Announcements 2/16/10 (posted with class notes)

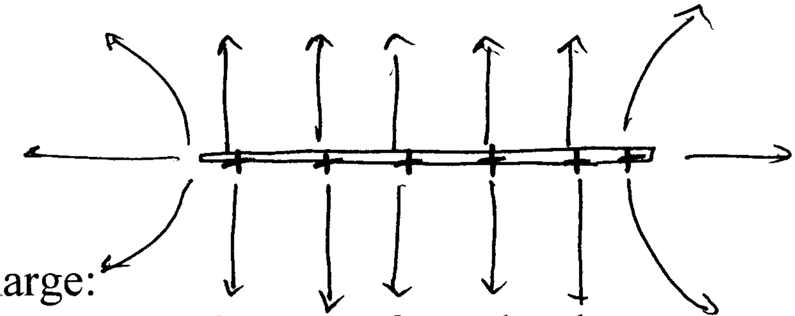
Midcourse feedback questionnaire on web site under “Reading”, due Thursday 2/18/10 at 11 a.m., worth 10 points of homework credit. You can finish it in just a couple of minutes by just answering multiple choice questions and answering “none” for the free-response, but if you want to give more detailed feedback that is welcome!

Last question on feedback questionnaire: If you have suggestions about topics you’d like to review a little in class on Thursday, I need those by Wednesday afternoon ...

Solutions to self-test problems are included in the problem set solutions, though sometimes they appear later. PS 2 and PS 3 solutions now include the self-test solutions.

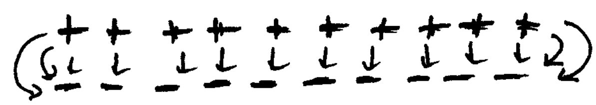
Reading for today: **22.2**, omitting “Continuous charge distributions” and Ex. 22.6 and 22.7, and **22.3** only up through “Got it?” 22.6. We will also cover **22.4** but only the first paragraph.
Thursday: 23.1-23.2

Key ideas from last time



Field of very large sheet of charge:
near the center of the sheet and not very far away from the sheet, the electric field is uniform and points perpendicular to the sheet everywhere

Two sheets of charge with equal and opposite charge density $\pm\sigma$ give uniform field between and zero field outside — like biological cell membrane



Dipoles feel zero net force in a uniform electric field, but a net force in a nonuniform electric field

Potential difference: change in electric potential energy per charge moving from location A to location B

$$\Delta V_{AB} = \frac{\Delta U_{AB}^{E}}{q} = \frac{-W^{E}}{q}$$

and for a uniform electric field,

$$\Delta V_{AB} = -\vec{E} \bullet \Delta\vec{r}_{AB} = -E\Delta r_{AB} \cos\theta$$

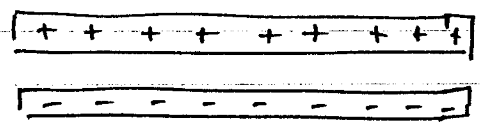
J.J.

2/16/2010

Today:

1. Potential difference in nonuniform field
2. Potential
3. Equipotential surfaces
4. Conductors in "electrostatic equilibrium" (steady $\vec{E}$) ~~no batteries attached~~

Begin with cell membrane potential difference problem from last time – membrane is like two sheets of charge



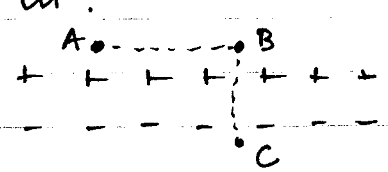
$$\Delta V_{out\ in} = -70 \text{ mV} = -70 \times 10^{-3} \text{ V}$$

What is strength of $\vec{E}$?

$$\Delta V_{out\ in} = -\vec{E}\cdot\Delta\vec{r}_{out\ in} = -E\,\Delta r_{out\ in}$$ b/c $\vec{E}$, $\Delta\vec{r}_{out\ in}$ in same direction

$$\Rightarrow E = \frac{-\Delta V_{out\ in}}{\Delta r_{out\ in}} = -\frac{(-70\times10^{-3}\text{V})}{7\times10^{-9}\text{m}} = 1\times10^{7}\ \text{N/C}$$

Does it matter where we choose to measure $\Delta V_{out\ in}$ from out to in?



Is ΔV_{AC} different from ΔV_{BC}?

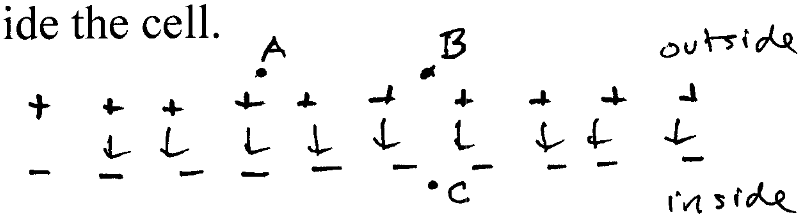
No: $\Delta V_{AC} = \Delta V_{AB} + \Delta V_{BC}$

and $\Delta V_{AB} = 0$ b/c $\vec{E} = 0$ outside the sheets

$$\Rightarrow \Delta V_{AC} = \Delta V_{BC}$$

Or: $\vec{E}\cdot\Delta\vec{r}_{AC} = \vec{E}\cdot\Delta\vec{r}_{BC}$ b/c component of $\Delta\vec{r}_{AC}$ $\parallel$ to $\vec{E}$ is same

Find the strength of the electric field in a cell membrane 7 nm thick with a resting potential difference of -70 mV from outside to inside the cell.



$$\Delta V_{out\ in} = -\vec{E} \circ \Delta\vec{r}_{out\ in} = -E\Delta r_{out\ in}$$

$$\Rightarrow E = -\frac{\Delta V_{out\ in}}{\Delta r_{out\ in}} = -\frac{(-70\times10^{-3}\,V)}{7\times10^{-9}\,m} = 1\times10^{7}\ N/C$$

$$\Delta V_{AC} \stackrel{?}{=} \Delta V_{BC}$$

(can I pick any point outside and any point inside to find $\Delta V_{out\ in}$?)

yes because $\vec{E} = 0$ outside and inside,

$$\Delta V_{AB} = 0$$

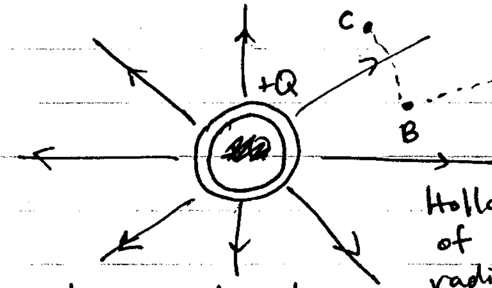
$$\Delta V_{AC} = \Delta V_{AB} + \Delta V_{BC}$$ path independence

~~Ways to calculate~~

Start w/ example of cell membrane

Potential and potential difference in nonuniform field

Still define potential diff as



Hollow shell of charge $+Q$ radius R

$$\Delta V_{AB} = \frac{\Delta U^E_{AB}}{q} = -\frac{W^E_{AB}}{q}$$

If $\vec{E}$ is not uniform, electric force is not constant.

→ must think of adding up work done by force on q as take very short steps from A to B

$$\Delta V_{AB} = -\frac{W^E_{AB}}{q} = -\frac{\displaystyle\sum_{\substack{\text{little steps}\\ \text{from A to B}}} \cancel{q}\vec{E}\circ\Delta\vec{r}_{step}}{\cancel{q}} = -\sum_{\substack{\text{little}\\ \text{steps}}} \vec{E}\circ\Delta\vec{r}_{step}$$

This is what an integral is:

$$\Delta V_{AB} = -\int_{r_A}^{r_B} \vec{E}\circ d\vec{r}$$

For each of our four benchmark arrangements of charge, we will find an expression for ΔV_{AB}

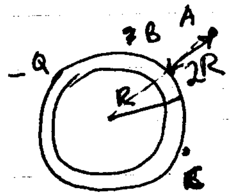
$$\Delta V_{AB} = -\int_{r_A}^{r_B} \frac{kQ}{r^2}\,dr = -\left(-\frac{kQ}{r}\right)\Bigg|_{r_A}^{r_B} = kQ\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

field of sphere <u>outside</u> the sphere

as long as $r_A, r_B \geq R$

Potential difference is path independent – only depends on endpoints – so can choose any convenient path.

CT

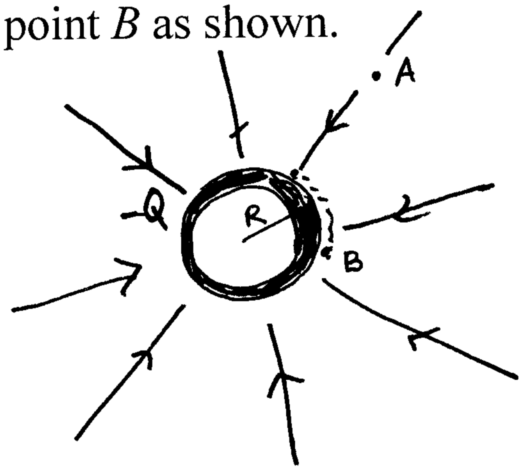


ΔV_{AB} is negative; $\Delta V_{BC} = 0$

Calculate: $\Delta V_{AB} = k(-Q)\left(\frac{1}{R} - \frac{1}{3R}\right) = -\frac{2kQ}{3R}$

(~~sample problem~~ instead)

An electron (charge –e) moves near a negatively charged hollow conducting sphere (total charge –Q, radius R) from point *A* to point *B* as shown.



path from A to B is in same direction as $\vec{E}$ up to surf sphere

$\vec{E} \cdot d\vec{r}$ is ⊕ everywhere (moving in same direction as $\vec{E}$)

$\Rightarrow W^E > 0$ on ⊕ particle

The potential difference ΔV_{AB} from point *A* to point *B* is

$\Rightarrow \Delta V_{AB_{surf.}}$ ⊖

1. positive
2. negative
3. zero.
4. Need more information.

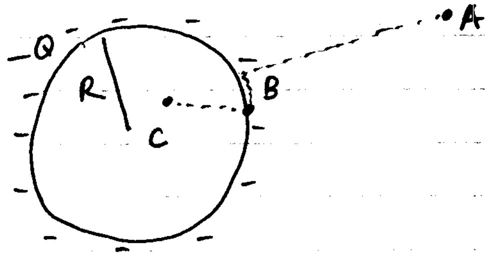
Doesn't depend on sign of charge being moved in $\vec{E}$, so can use ⊕

Work done on ⊖ electron!

force is opposite to motion $\Rightarrow W^E$ ⊖

$$\Delta V_{AB} = -\frac{W^E}{q} = -\frac{W^E}{-e} =$$ same sign as work on ⊖ particle

What about inside the sphere? What is ΔV_{AC}?



Again think of getting from A to C via B:

$$\Delta V_{A \text{ to surface}} = k(-Q)\left(\frac{1}{R} - \frac{1}{r_A}\right)$$

$$\Delta V_{\text{surface to } B} = 0 \quad \text{b/c} \perp \text{ to } \vec{E} \text{ everywhere}$$

To find ΔV_{BC} need to know $\vec{E}$ inside sphere.

Turns out inside a hollow charged shell, $\vec{E}=0$;
inside fields from all parts of shell cancel

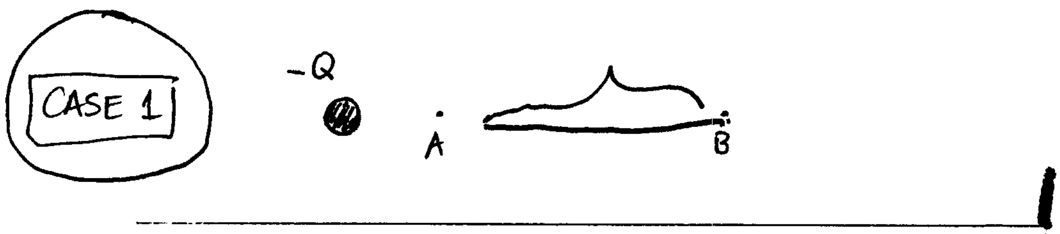
$$\Rightarrow \quad \Delta V_{BC} = 0 \text{ because if } \vec{E}=0, \quad W^E_{BC} = -\int_B^C \vec{E}\cdot d\vec{r} = 0!$$

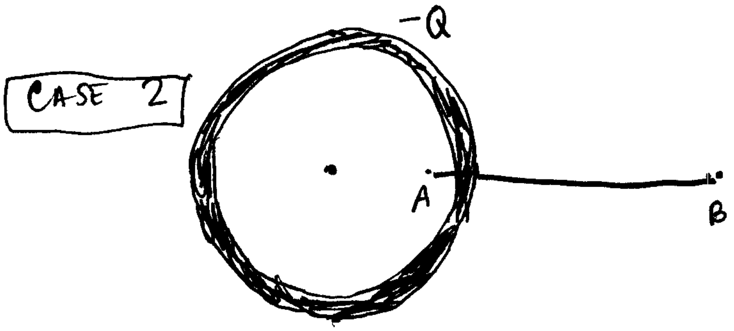
Potential difference is zero if electric field is zero in region between two points.

[Potential is constant!]

12:50 $\boxed{\text{CT}}$ Compare: which has greater $|\Delta V_{AB}|$?

In which case, Case 1 or Case 2, is the magnitude of the potential difference $|\Delta V_{AB}|$ greater?





Divide up path from ~~B~~ A to B in two parts

$$\Delta V_{A\,\text{shell}} + \Delta V_{\text{shell}\,B} = \Delta V_{AB}$$

0

compare to ΔV_{AB} in case 1

$\vec{E}$ outside shell is same as $\vec{E}$ outside ball w/ Q (case 1)

Potential:

only changes in potential energy ΔU^{E}_{AB} are physically meaningful

Just as with gravity, we are free to choose zero wherever convenient

Potential $V(r)$ means potential from some chosen reference point

in notation analogous to ΔV_{AB} would write as $\Delta V_{ref\ r}$

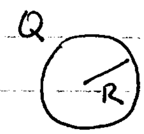
For individual point charges or spherical: choose reference ∞ far away where $\vec{E}=0$, so no energy

for sphere or point charge

$$\Rightarrow \quad V(r) = kQ\left(\frac{1}{r_B} - \frac{1}{r_A}\right) \quad \text{with } r_B = r, \; r_A = \infty$$

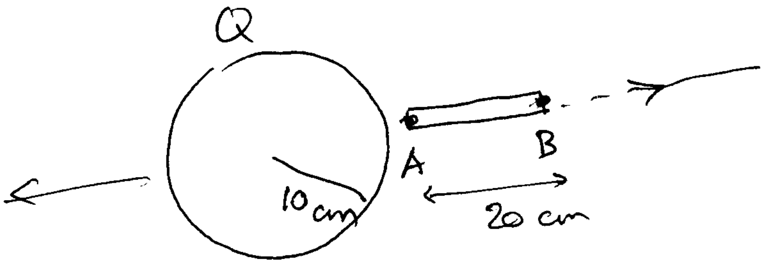
So potential is

$$\Rightarrow \quad V(r) = \frac{kQ}{r} \quad \text{for } r > R$$



Potential of surface of sphere is $V(r=R) = \dfrac{kQ}{R}$

The light bulb lights up in the electric field of the van de Graaf generator when there is a potential difference of 110 V from one end to the other. The bulb is 20 cm long and the dome of the generator is approximately spherical with radius approximately 10 cm. If the bulb is held as close as possible to the dome without touching, what minimum charge must be on the generator dome for the bulb to light up?



hold parallel to field
→ greatest ΔV between ends
hold close to generator
→ strongest $\vec{E}$ → greatest ΔV

$\Delta V_{AB} = 110\text{V}$ want Q to give this

given $r_A = 0.10\text{ m}$
$r_B = 0.30\text{ m}$

$$\Delta V_{AB} = kQ\left(\frac{1}{r_B} - \frac{1}{r_A}\right) \Rightarrow Q = \frac{\Delta V_{AB}}{k\left(\frac{1}{r_B} - \frac{1}{r_A}\right)}$$

sub in values $\Rightarrow Q = 1.8 \times 10^{-9}\text{C}$

DEMO

Problem: light bulb in $\vec{E}$ of van de Graaf:
→ hold bulb so it is ~~radially~~ aligned along a radial line → greatest potential difference between ends ←

hold bulb w/one end nearly touching — field is strongest ~~[illegible]~~ here so ΔV will be greatest ($\int \vec{E} \cdot d\vec{\ell}$ is greatest where $\vec{E}$ is strongest)

need $\Delta V_{AB} = 110V$, want Q

$r_A = 10$ cm

$r_B = 10 + 20$ cm $= 30$ cm

$$\Delta V_{AB} = kQ\left(\frac{1}{r_B} - \frac{1}{r_A}\right) \Rightarrow Q = \frac{\Delta V_{AB}}{k\left(\frac{1}{r_B} - \frac{1}{r_A}\right)} = 1.8 \times 10^{-9} C$$

This Q will produce the necessary ΔV_{AB}.

What if instead hold bulb w/ both ends @ same distance from vdG?

shouldn't light up — $\Delta V_{AB} = 0$!

Equipotentials

Lines/surfaces of constant potential: points that are all at the same potential energy

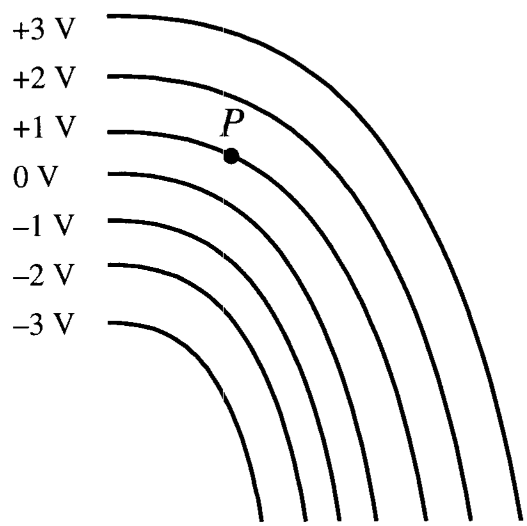
Analogy to gravity: surfaces of constant height above the earth are all at the same gravitational potential

Topographic map shows lines of constant altitude

~~Lines~~ Surfaces are $\perp$ to $\vec{E}$ because ~~[illegible]~~ then $\vec{E}$ does no work on a charge moving along such a surface

vdG dome: any points @ same distance from ctr are equip.

A small negative point charge is placed at point P in the electric field shown below. When the charge is released, if there are no external forces, in which direction will it initially move?

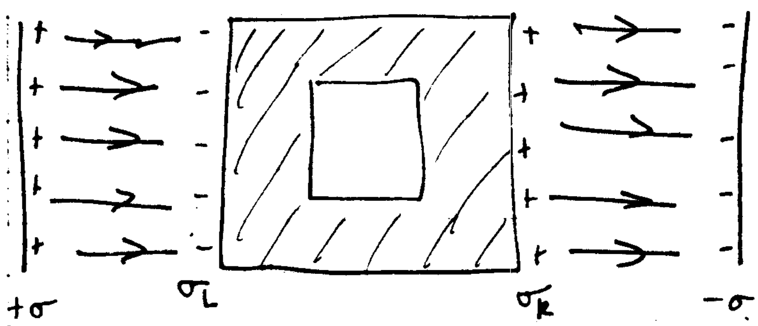


1. Along the 1-V equipotential.
2. Perpendicular to the equipotential lines, toward the 0 volt line.
3. Perpendicular to the equipotential lines, toward the 2 volt line.

Conductors in Electric fields

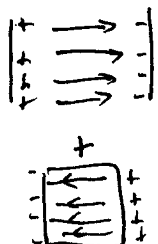
in a conductor some charges are completely free to move — they will move if there is any force on them

∴ they move until there is no $\vec{E}$ in the conductor anywhere except right at the surface



hollow metal box with no excess charge (total chg of box is zero)

"Electrostatic equilibr."



gives $\vec{E}=0$ in box

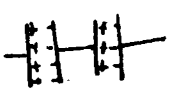
Polarizes to produce $\vec{E}=0$ in bulk of conductor

Only charge density is on <u>surface</u> – unbalanced charges are at surface, elsewhere charge is neutral

$\vec{E}$ must be $\perp$ to surface so that force on charge would take out

(Different from insulator that can only shift e^- a little)

$\boxed{\text{CT}}$ What is the charge ~~de~~ per area σ_R on the ~~left~~ right side of the box? $\sigma_R = +\sigma$ (to cancel field in metal)

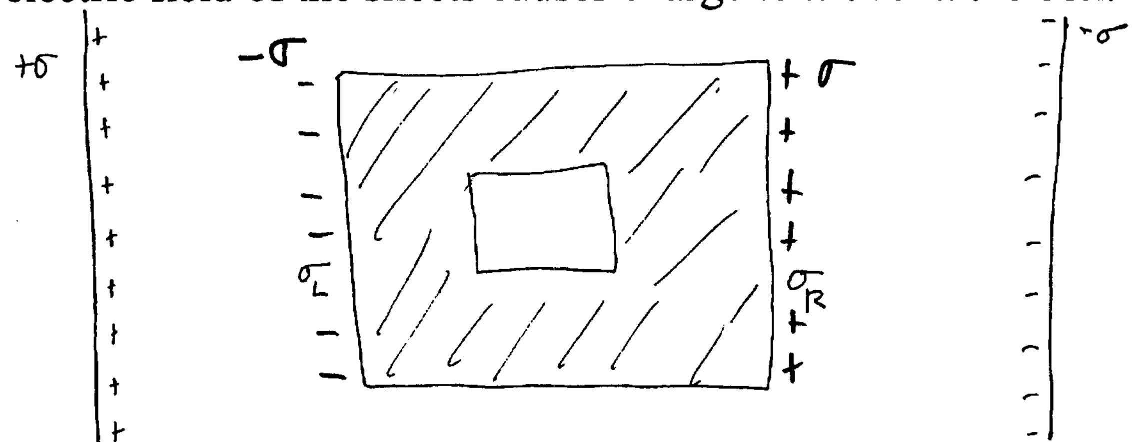
$\boxed{\text{CT}}$ What is $\vec{E}$ inside the box? zero: think  (any shape)

In general: $\vec{E}$ inside any empty hollow conductor is zero

Radio demo

Shielding – Crawddd manual, coax

A hollow conducting box is placed in the uniform field produced by two large charged sheets with charge $\pm\sigma$ as shown. The electric field of the sheets causes charge to move in the box.



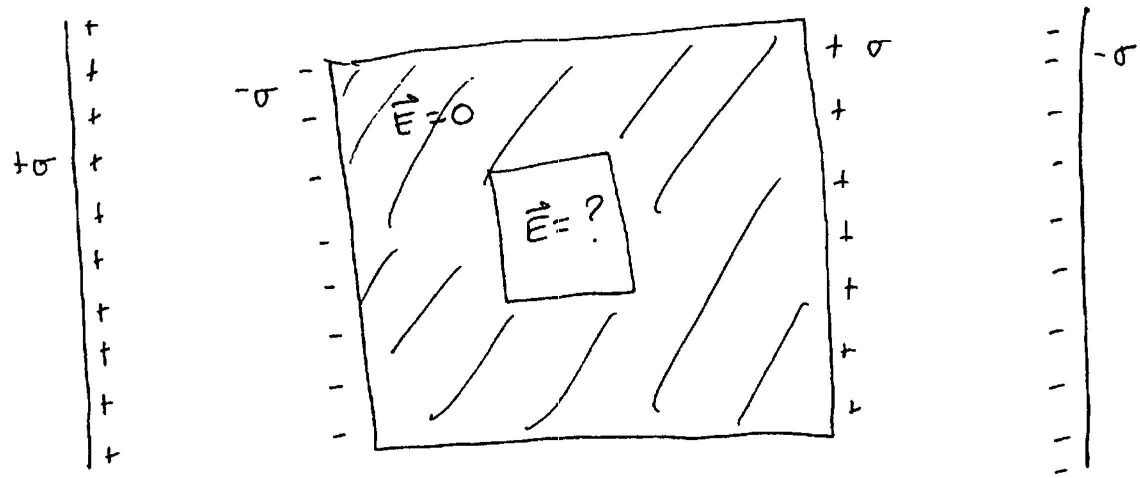
The surface charge density σ_R induced on the right surface of the box is

1. positive and less than $+\sigma$
2. positive and approximately equal to $+\sigma$
3. positive and greater than $+\sigma$
4. negative and less than $-\sigma$
5. negative and approximately equal to $-\sigma$
6. negative and greater than $-\sigma$
7. zero because the box does not have a net charge.

Total charge on box ("net charge") still zero

Only moved charge around – haven't added any to box

A hollow conducting box is placed in the uniform field produced by two large charged sheets with charge $\pm\sigma$ as shown. The electric field of the sheets causes charge to move in the box.



What is the electric field inside the hollow part of the box?

1. $4\pi k\sigma$ to the right.
2. $4\pi k\sigma$ to the left.
3. Zero.
4. Need more information.