Announcements 4/15/10

Added extra problem solution to last Thursday's lecture notes

Reading:
Today: 29.6-29.8
Next Tuesday: 32.1-32.2

Review of mathematics of sinusoidal waves tonight (same time as problem session)

Office hours Friday 3:30 - 5

Sinusoidal plane waves

Last time introduced these as simplest possible building block of traveling $\vec{E}$ & $\vec{B}$ wave: any pattern of EM fields can be constructed by combining waves of appropriate freq and amplitude

- transverse ($\vec{E} \perp \vec{B} \perp$ dir. of travel)
- in phase ($\vec{E}, \vec{B}$ max @ same time)
- depend only on x & t

Mathematically:

$$\left.\begin{aligned}\vec{E} &= E_p \sin(kx-\omega t)\,\hat{j} \\ \vec{B} &= B_p \sin(kx-\omega t)\,\hat{k}\end{aligned}\right\} \text{ travels in } +x \text{ direction}$$

$\sin(kx+\omega t)$ travels in $-x$ direction

Show Physlet to show moving pattern

$\boxed{CT}$ Rank $\vec{B}$

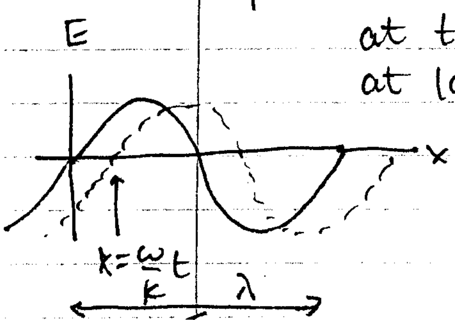
discuss how $\vec{E}$ ranking is same

Speed at which pattern travels is $v = \frac{\omega}{k}$

at $t=0$ have $\sin(kx)$

at later t have $\sin(kx-\omega t)$

starting point at $kx-\omega t=0$ has shifted to $x = \frac{\omega}{k}t = vt$



ω = freq of oscillation $= 2\pi f$ (ω in rad/s, f in Hz $= 1/s$)

to see this: think of sitting at one point in space, say $x=0$

at this point $\vec{E} = E_p \sin(-\omega t)\,\hat{j}$ period of osc $= \frac{1}{f} = T = \frac{2\pi}{\omega}$

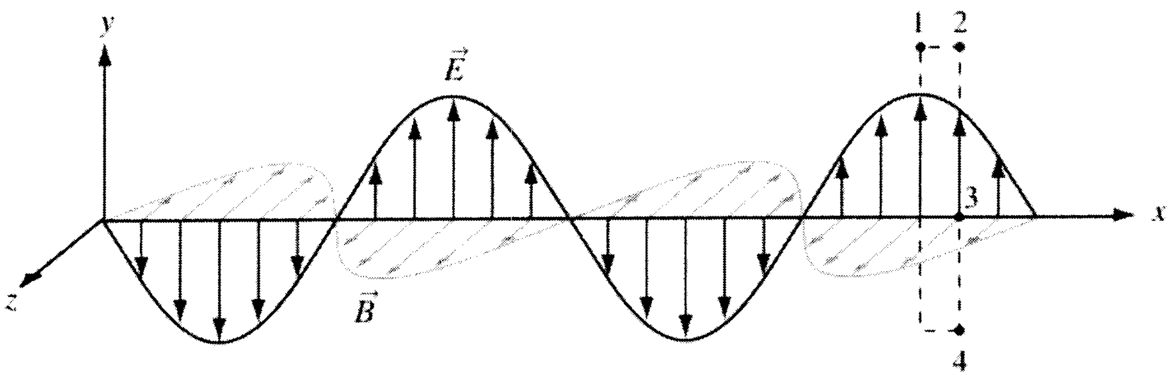
$\vec{B} = B_p \sin(-\omega t)\,\hat{k}$

k = "spatial frequency"; length of repeating pattern in space is λ, $k = \frac{2\pi}{\lambda}$

(at $t=0$ $\vec{E} = E_p \sin kx\,\hat{j}$, repeats at $kx = 2\pi$)

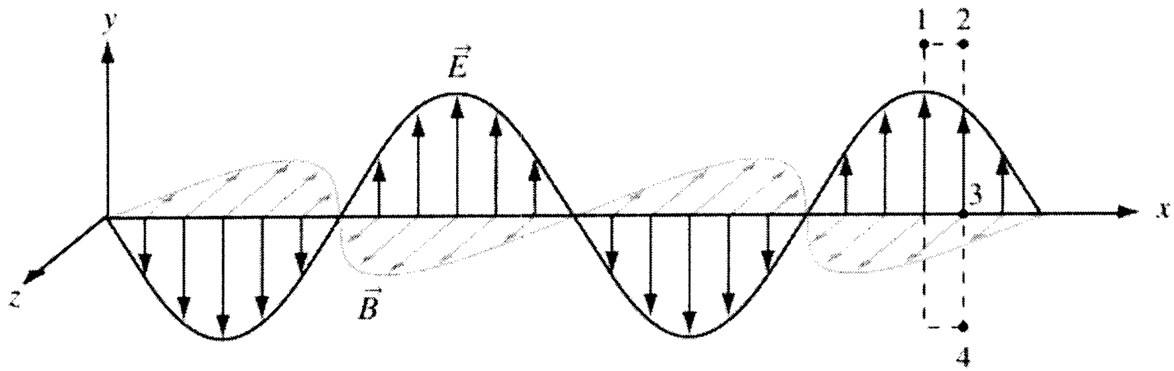
~~Amplitudes of $\vec{E}$ and $\vec{B}$ are related due to process of mutual induction:~~

The four numbered points on the figure below lie in the *x*-*y* plane. For the instant shown, rank the strength of the **electric** field at each of the four numbered points.



1. 1 = 2 > 4 >3
2. 1 = 2 > 3 = 4
3. 1 > 2 = 3 = 4
4. 1 > 2 > 3 > 4

The four numbered points on the figure below lie in the *x*-*y* plane. For the instant shown, rank the strength of the **magnetic** field at each of the four numbered points.



1. The magnetic field at all four points is zero.
2. The magnetic field is the same at all four points but it is not zero.
3. 1 = 2 > 4 > 3
4. 1 = 2 > 3 = 4
5. 1 > 2 = 3 = 4
6. 1 > 2 > 3 > 4

Similarly amplitudes of $\vec{E}$ and $\vec{B}$ are related due to process of mutual induction:

$$B_p = \frac{E_p}{c}$$

and thus at <u>any</u> instant $B = \frac{E}{c}$ relates field strengths

Speed of EM waves

Mechanical waves (water, springs):
v is a complex function of ω and k
depends on material in which wave travels ("medium")

EM waves in vacuum: $v = c$ independent of ω, k!
in fact, this is how physicists figured out light was an EM wave: diff. eqs. for $\vec{E}$ and $\vec{B}$

$$\mathcal{E} = \int \vec{E}_{ind} \circ d\vec{r} = -\frac{d\Phi_B}{dt}$$

$$\int \vec{B}_{ind} \circ d\vec{r} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

gives coupled diff eqs for $\vec{E}$ and $\vec{B}$

Solving these gives an equation that any wave of $\vec{E}$ & $\vec{B}$ must satisfy and predicts it would travel at

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

measure μ_0 & ϵ_0 from circuits
measure v from travel of light signals: $c = 3.00 \times 10^8$ m/s
Match perfectly!

As a result, simple relationship between ω and k:
$\omega = ck$ with $c = 3.00 \times 10^8$ m/s
Usually write it in terms of f and λ
$\Rightarrow c = f\lambda$

Changing f or λ changes the color / type of EM radiation: show figure (study on your own)

~~high f / short λ = high energy per photon~~
~~low f / long λ = low energy per photon~~

f and λ affect how they interact with matter — absorbed or transmitted

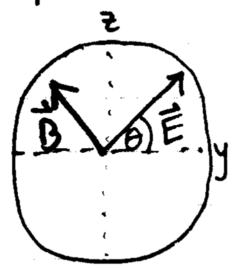
Polarization

We are mostly interested in what happens to light waves when they interact with objects — i.e. when they are reflected, transmitted, or absorbed!

Because electric interactions are much stronger than magnetic for most materials, these interactions are dominated by $\vec{E}$ affecting the charges in the material

⇒ Give a special name to direction of $\vec{E}$: call it the direction of <u>polarization</u>

Represent wave traveling toward you as follows:



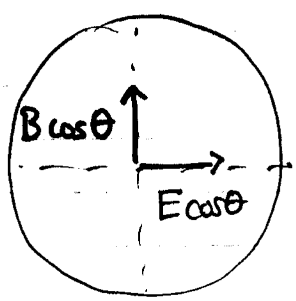
draw vector showing direction of $\vec{E}$ at some instant

ask: which way is $\vec{B}$?

polarization here is at angle θ to y-axis

Just as with all other $\vec{E}, \vec{B}$: can decompose fields into components: this wave is the sum of waves

recall $B = \frac{E}{c}$



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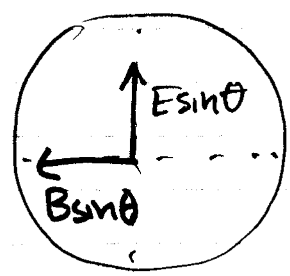
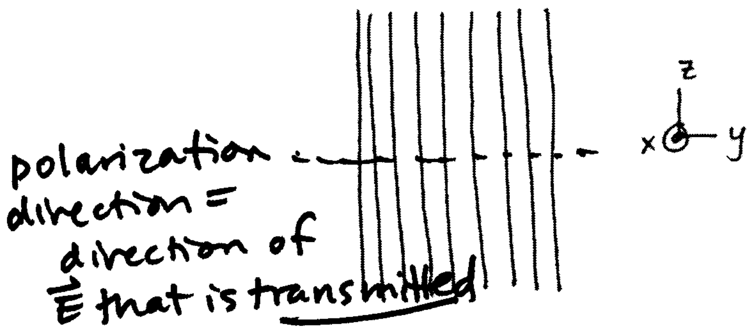


horizontal polarization   vertical polarization

Light from light bulbs, sun, etc is unpolarized — random mixture of waves of all possible polarizations

A plane electromagnetic wave traveling in the +x direction reaches the set of parallel conducting wires shown. Which wave causes current to flow in the wires?



1. $\vec{E} = E_p\hat{j}\sin(kx-\omega t)$ and $\vec{B} = B_p\hat{k}\sin(kx-\omega t)$
2. $\vec{E} = E_p\hat{k}\sin(kx-\omega t)$ and $\vec{B} = -B_p\hat{j}\sin(kx-\omega t)$
3. Neither wave will cause current to flow.
4. Both will cause current to flow.

Polarizing filter

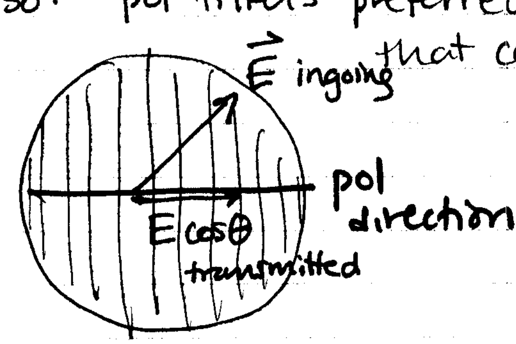
object that only allows EM waves of a particular polarization (the "preferred direction" or "polarization direction" of the filter) through

So – if hold filter with its pol direction horizontal, only the horiz pol is passed through
Works by absorbing $\vec{E}$ that is $\perp$ to pol direction

How does it do this?
Molecular structure is long parallel strands of molecules that can conduct current

$\boxed{\text{CT}}$ Which wave causes current to flow?
The one with $\vec{E}$ along the molecules — get an oscillating current

Energy is absorbed — current dissipates power (I^2R)
$\Rightarrow$ this wave is absorbed!
The wave with $\perp$ $\vec{E}$ does <u>not</u> cause much current
$\Rightarrow$ not absorbed
So: pol filter's preferred direction is $\perp$ to the molecules that can conduct

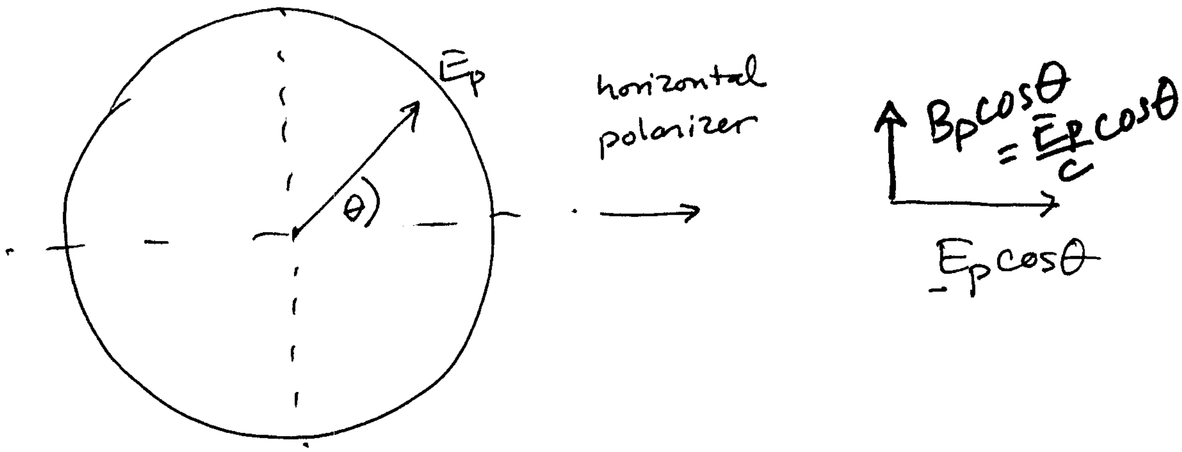


EM wave that comes in at an angle: only the part along the pol direction is transmitted
Show simulation?

could skip

$\boxed{\text{CT}}$ Transmitted $\vec{B}$?
vertical, magnitude $B\cos\theta$
$\vec{B}$ induced by $\vec{E}$ and vice versa — amplitude of $\vec{B}$ must always match amplitude of $\vec{E}$

A plane wave with electric field amplitude E_p and polarization at angle θ to the horizontal passes through a horizontal polarizer. What is the amplitude of the magnetic field of the wave after passing through the polarizer?



1. E_p/c
2. $E_p\cos\theta/c$
3. $E_p\sin\theta/c$

Energy in EM waves

We all know light carries energy (sunburn, dark objects heating up on sunny day) - where is that energy?

Previously: energy stored in capacitor - thought of as coming from arrangement of charge
$U_{cap} = \frac{1}{2} C \Delta V_{cap}^2$

Can also think of energy as stored in $\vec{E}$

$$u_E = \frac{\text{energy}}{\text{volume}} = \frac{1}{2}\epsilon_0 E^2 \quad \text{density of energy in } \vec{E}$$

Likewise $\vec{B}$ stores energy:

$$u_B = \frac{\text{energy}}{\text{volume}} = \frac{1}{2\mu_0} B^2$$

As pattern of $\vec{E}$ & $\vec{B}$ moves, carries with it the energy in those fields - want to know power = rate at which energy is delivered

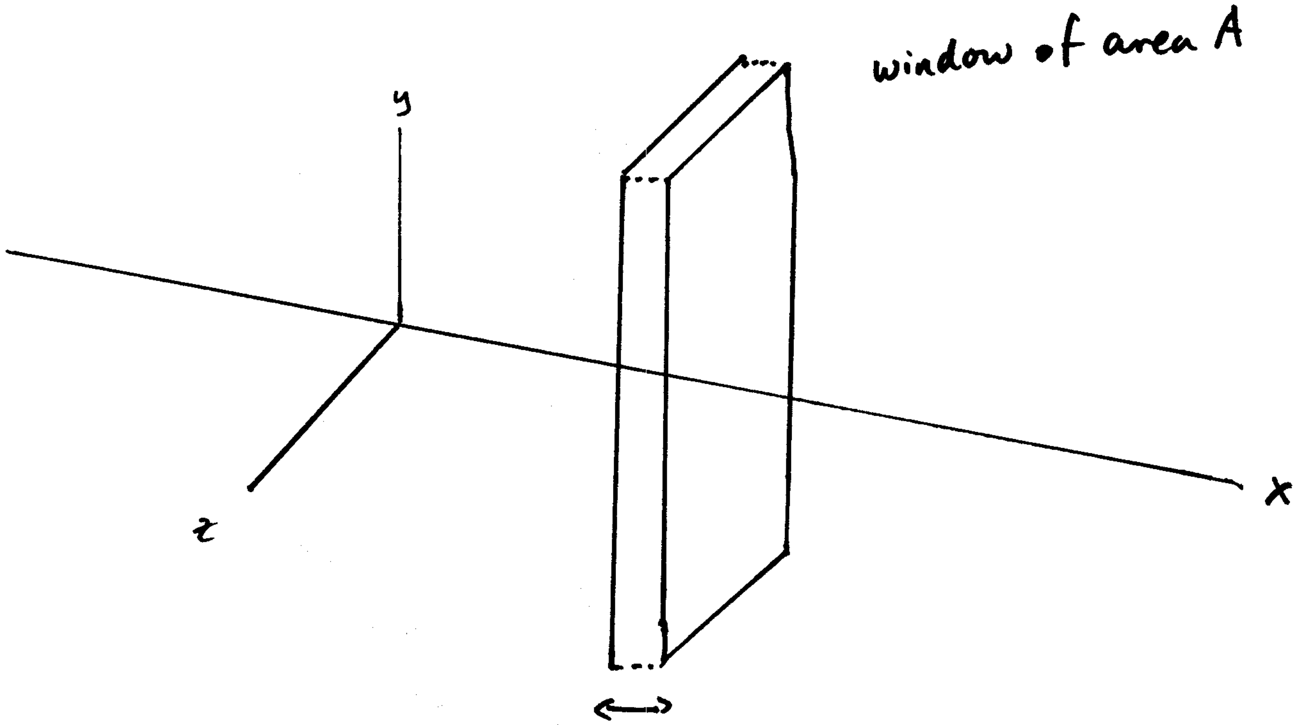
Typically light waves spread out over some area so we are ~~most~~ often interested in a quantity called <u>intensity</u>

$$\text{intensity} = \frac{\text{power}}{\text{area}} \qquad \text{units: } \frac{W}{m^2}$$

Vector $\vec{S}$ is intensity with direction = direction of wave travel

Find intensity of plane wave in terms of E and B (go to handout): intensity $\propto E^2$!!

[could do CT here]



$U \equiv$ energy stored in EM wave fields

$$\underset{\text{in window}}{dU} = (\text{volume})\left(\frac{\text{energy}}{\text{volume}}\right) = A\,dx\left(\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2\right)$$

All energy travels through in $dt = \frac{dx}{c}$

$$\Rightarrow \quad \frac{dU}{dt} = Ac\left(\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2\right) = \text{power}$$

$$\text{Intensity} = \frac{\text{power}}{\text{area}} \quad \Rightarrow \quad S = \frac{c}{2}\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right)$$

use $B = \frac{E}{c}$ and $c^2 = \frac{1}{\mu_0 \epsilon_0}$ to rewrite:

$$S = \epsilon_0 c E^2 \quad \text{or} \quad S = \frac{EB}{\mu_0}$$

$\boxed{\text{intensity} \propto E^2 !}$

Define vector intensity to point in direction of travel:

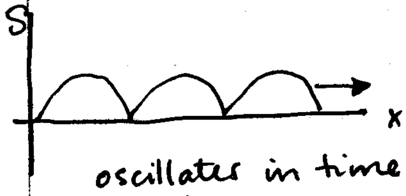
direction is $\vec{E} \times \vec{B}$

because $\vec{E} \perp \vec{B}$, magnitude of $\vec{E} \times \vec{B}$ is just EB: so can write intensity vector as

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} \qquad \text{"Poynting vector"}$$

All of this is in terms of the oscillating fields: $E = E_p \sin(kx - \omega t)$

means $S \propto \sin^2(kx - \omega t)$



oscillates in time and space

Always positive! means energy is always traveling forward with wave (if S was sometimes $\ominus$ would be like losing energy then)

Oscillations are very fast: radio is MHz

visible light is 10^{15} Hz

so usually we want average intensity over time

Average over time of $\sin^2(kx - \omega t) = \frac{1}{2}$

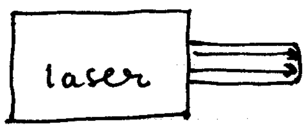
so $S = \epsilon_0 c E_p^2 \sin^2(kx - \omega t)$

gives $\overline{S} = \frac{1}{2} \epsilon_0 c E_p^2$

[Average over time of $\sin(kx - \omega t)$ is zero: so usually want peak value of E]

Plane wave is an idealization — but it's a good one for
two types of sources of light

1. Beamlike light source: laser beam, flashlight
Light is uniformly traveling in a single direction and $\vec{E}, \vec{B}$ the same throughout the area of the beam

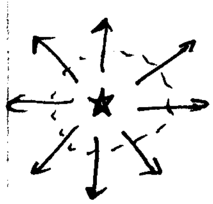


within beam, can treat it like an ideal plane wave

avg intensity does not depend on distance from laser —

$$S = \frac{\text{power}}{\text{area}} = \frac{\text{total power of laser}}{\text{area of beam}}$$

2. Point source of light that is far away: sun, star
Point sources produce waves that spread out in all directions: think back to optics!



each ray represents direction of travel of EM wave

wave fields oscillate along each ray (draw in)

Total power of source therefore spreads out over spherical surfaces centered on star

$$S(r) = \frac{\text{power}}{4\pi r^2}$$

distance from source

surface area of sphere

E and B likewise decrease in amplitude: $\frac{1}{r^2}$

still have $S(r) = c\epsilon_0 E_p^2(r)$

Cell phone tower, light bulb: if given total power of source, can find intensity at some distance from the source

If very far away: changing ~~B~~ r a little bit doesn't affect things much — waves don't spread much, S constant