Announcements 3/4/10

Thursday evening after break (3/18/10): review of circular motion and cross products. (We'll start using these ideas in class that day.)

Handout: PS6. A couple of additional capacitor problems included plus a dipole problem related to this week's ECG analysis.

Reading:
Today: 25.1-25.2. I
Lab prep week of 3/15: 24.5 and 25.4 (plus a problem)
Tuesday 3/16: 25.3 and 25.5

Tuesday's class notes include what students contributed about SDS-PAGE protein preparation.

Feedback questionnaire posted tomorrow (in reading area again), complete any time over break for homework extra credit.

Still grading exams, will email when they are ready. Feel free to request your score by email if you are away for break.

J.J.

3/24/2010

Circuit: way to convert electrical energy in battery/source of emf to some useful form

$U^E \rightarrow$ heat: toaster, incandescent light bulb
$U^E \rightarrow$ light: fluor light bulb
$U^E \rightarrow$ motion: motor, proteins in gel, muscle fiber contr.
$U^E \rightarrow$ signal: nerve impulse, radio trans, computer

Big picture: how are voltage and current, and hence energy, distributed in circuits – how much energy goes where?

1. Define circuit, introduce symbols & standard approx.
2. Three principles of ckt analysis
   - Conservation of energy
   - Conservation of charge
   - $V = IR$
3. Simple circuit building blocks: series ckt ("voltage divider")
   parallel ckt ("current divider")
4. Multiloop circuits w/one battery
5. Multiple batteries

prob. after break

Key ideas from last time

Source of emf (battery, power supply):
converts non-electrical energy (usually chemical) to U^E to maintain potential difference $\mathcal{E}$ between terminals

Microscopic picture of current
Defined current: $I = \Delta Q/\Delta t$
positive current: either + charge moving in direction of $\vec{E}$ (toward lower potential) or – charge going opposite way

Amount of current: $I = nqAv_d$
density n and charge q of mobile charges
cross-sectional area through which charges are traveling

Average speed v_d ("drift speed"): $v_d = \frac{q}{m}E\tau$

increases with electric field strength E

How much current do you get for a given potential difference across a conductor?
In most conductors ("ohmic"), current is proportional to E; gives
$\Delta V_{cond} = IR$ with $R=\rho L/A$
resistivity ρ: property of the particular material
related to n, τ, q/m

How fast does energy get converted to heat?
$P = I\Delta V_{cond}$

You are running current through a gel that is 10 cm tall, 10 cm wide, and 1 mm thick, filled with 0.01 mol/L NaCl solution (6.023×10^{24} ions/m^3). The voltage from the top to the bottom of the gel is 110 V and the current in it is 100 mA. Find the drift speed of the ions in the gel.

$$I = I_+ + I_-$$ (100 mA is partly from Na^+ partly from Cl^-)

Approximate drift speeds of Na^+ and Cl^- as equal

$$\Rightarrow I_+ = I_- = \frac{1}{2}(100\ \text{mA}) = 50\ \text{mA}$$

$$I_+ = nqAv_d$$

(Substitute values from problem:

$n = 6.023 \times 10^{24}$ ions/m^3

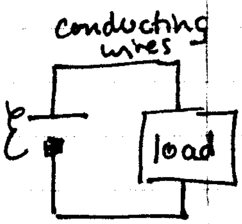
q = elementary charge $= 1.6 \times 10^{-19}$ C

A = 10 cm × 1 mm = $(0.10\text{m})(0.001\text{ m})$

$$\Rightarrow v_d = \frac{I_+}{nqA} = \frac{50 \times 10^{-3} \text{A}}{(6.023 \times 10^{24} \text{ ions/m}^3)(1.6 \times 10^{-19} \text{C})(1 \times 10^{-4} \text{m}^2)}$$

$$= 5.2 \times 10^{-4} \text{ m/s}$$

Circuit: connect source of emf and "load" in a closed loop around which current can flow



"load": whatever you want to take U^E and convert it to sthg else

Load always has significant resistance R
means that as current flows through it,
there must be a corresponding ΔV_{load} : call it V_{load}

potential difference across load

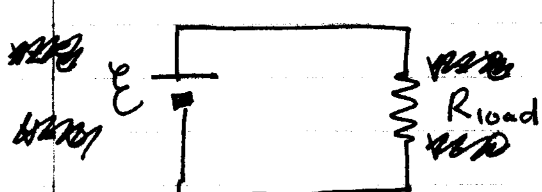
Standard to get rid of the deltas for convenience
Remember that in ckts this really means ΔV_{load} !!

Likewise this means energy delivered to it: remember Tuesday

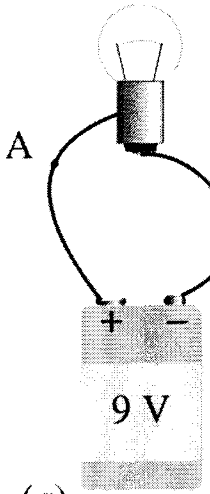
$$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

We make approx that connecting wires have $R=0$
$\Rightarrow$ even though there is current in them,
$V_{wires} = 0$ and $P_{wires} = 0$
Thus these wires are equipotentials.
Pretty good approx b/c as you'll see in lab, $R_{wires} \ll R_{load}$

So (show ckt figure) means $V_{load} \approx \mathcal{E}$. $\boxed{\text{Ref for zero is } \ominus \text{ term of batt.}}$
- end of load that is connected to $\oplus$ of batt is at $\mathcal{E}$
- end of load connected to $\ominus$ of batt is at 0.



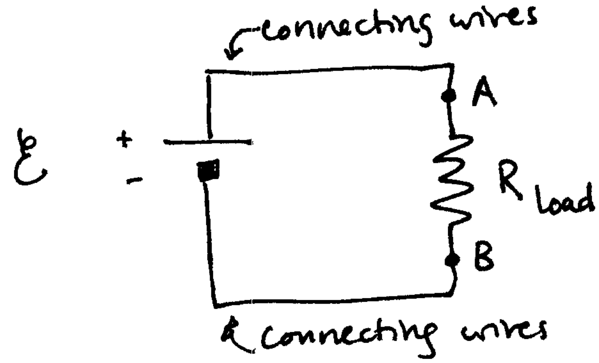
Circuit with a single load:



(a)

Bulb filament is a conductor with resistance

→ represent w/circuit diagram



Approximate $R_{wires} \approx 0 \Rightarrow \Delta V_{wires} = 0$ also

Then potential at $A = \mathcal{E}$ } taking reference $V=0$ at ⊖ batt terminal "ground"

** potential at $B = 0$ "ground"

Means $\Delta V_{AB} = -\mathcal{E}$

Convention:

Represent potential diff across load ΔV_{load} as V_{load} (leave out Δ) – implied difference

Also choose $V_{load} = |\Delta V_{AB}|$ absolute value : lets you use $V=IR$

keep track of signs separately in circuit analysis

so $V_{load} = IR$ means potential diff across load $= \begin{pmatrix}\text{current} \\ \text{in load}\end{pmatrix} \times \begin{pmatrix}\text{resist} \\ \text{of load}\end{pmatrix}$

call it the "voltage across the load"

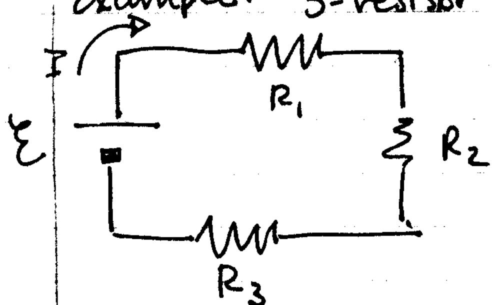
[Repeat principles on board] ~~examples~~

Three principles of circuit analysis:

initially write on doc cam

1. Conservation of energy ~~= potential~~ ~~energy diffs~~ ~~differences~~
   potential energy changes for a charge traveling around a closed loop add to zero
   → potential differences sum to zero around a loop
   "loop rule"

Example: 3-resistor ckt



$$q\mathcal{E} - qV_{R_1} - qV_{R_2} - qV_{R_3} = 0$$

charge gains energy $q\mathcal{E}$ in batt — loses energy qV_{R_1} in R_1 .... — back to original energy

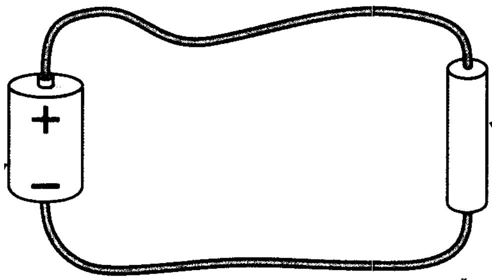
$$\Rightarrow \quad \mathcal{E} - V_{R_1} - V_{R_2} - V_{R_3} = 0$$

2. Conservation of charge
   charge does not get used up or accumulate anywhere (if there ~~[illegible]~~ are only closed loops)
   - in a single-loop circuit current is constant around the loop
     (which way does it flow? as shown)
   - at a junction, total current in = total current out

$\boxed{\text{CT}}$ Current inside battery.

3. Ohm's law: $V_R = I\,R$
   V_R: ~~[illegible]~~ voltage across resistor
   I: current in resistor
   R: resistance

Write principles onto blackboard



Which way do mobile positively charged ions move *inside* the battery?

1. From the negative terminal to the positive terminal.
2. From the positive terminal to the negative terminal.
3. Some move in each direction so there is no net motion of charge through the battery.

(Skipped; answer is from − to + b/c battery gives energy to charges, and thus moves them against $\vec{E}$, in direction of increasing energy.)

$\boxed{\text{CT2}}$ which resistor has greatest voltage? V_{R_3}

Principle #2: current is same all around

Principle #3: apply Ohm's law: $V_R = IR$ so largest R $\rightarrow$ largest V

Problem: find current in circuit

$\boxed{\text{Principle \#1}}$:

$$\mathcal{E} - V_{R_1} - V_{R_2} - V_{R_3} = 0$$

$\boxed{\#3}$
$$\mathcal{E} - IR_1 - IR_2 - IR_3 = 0$$

Solve for I: $I = \dfrac{\mathcal{E}}{R_1+R_2+R_3} = \dfrac{9.0\text{ V}}{4.5\text{ k}\Omega + 4.5\text{ k}\Omega + 9.0\text{ k}\Omega}$

$$I = 0.5\ \frac{\text{V}}{\text{k}\Omega} = 0.5 \times 10^{-3}\text{ A} = 0.5\text{ mA}$$

Then to find individual voltages use value for current

Notice this can be written

current in battery is same as current elsewhere

$I_{batt} = \dfrac{\mathcal{E}}{R_{eq}}$ with $R_{eq} = R_1 + R_2 + R_3$ usually called "equivalent resistance"

Same current as if 1 resistor of resistance R_{eq} connected!

Series ckt called "voltage divider" b/c voltage gets spread over individual resistors

Notice $V_{R_1} = IR_1 = \dfrac{\mathcal{E}}{R_{eq}} R_1 = \mathcal{E}\left(\dfrac{R_1}{R_{eq}}\right)$

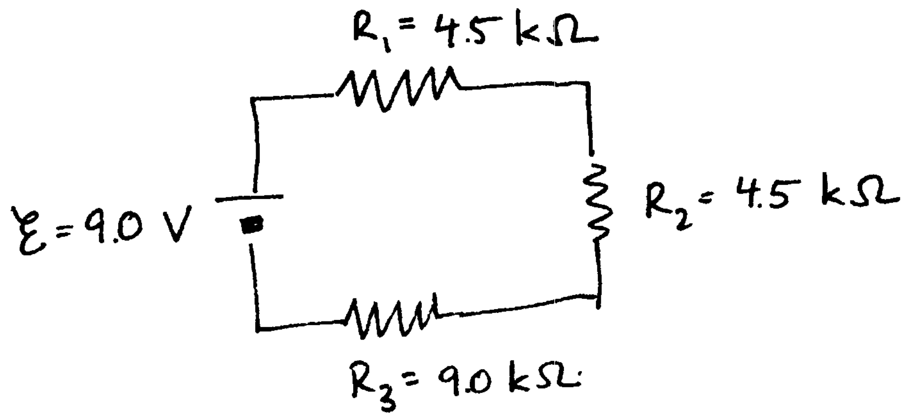
fraction of R_{eq} that R_1 is

What happens to current as add resistors in series?

Decreases b/c R_{eq} increases: Demo

Brightness of bulb $\propto P = I^2R = V^2/R$ and $I = \dfrac{\mathcal{E}}{R_{eq}}$

In the circuit below, across which resistor is the voltage (potential difference) greatest?



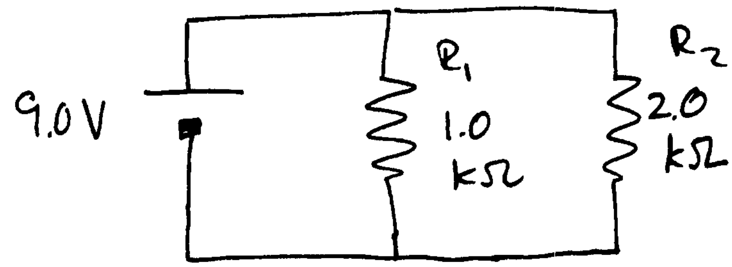
1. R_1
2. R_2
3. R_3
4. The voltages across R_1 and R_2 are equal and greater than the voltage across R_3.
5. All three are the same.
6. Need more information.

$V_R = IR$

I same throughout

$\rightarrow$ biggest V is across largest R

In the circuit below, across which resistor is the voltage (potential difference) greatest?

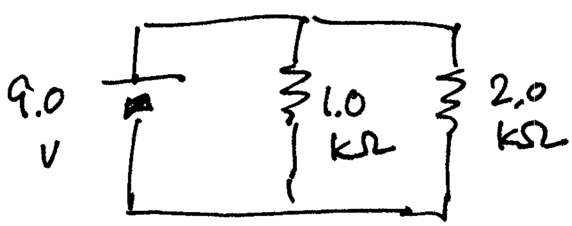


Both resistors are connected at top and bottom to battery

"parallel" – connected to same point in ckt above & below

1. 1.0 kΩ
2. 2.0 kΩ
3. The voltages across both are the same.
4. Need more information.

Which resistor has greater current?

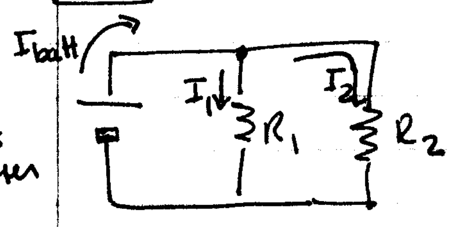


1. 1.0 kΩ
2. 2.0 kΩ
3. Same

Branching circuit: think about how loop rule applies

$\boxed{\text{CT}}$ Parallel: <u>same</u> voltage

Don't add currents until after CT



loop rule:

left loop $\mathcal{E} - V_{R_1} = 0$

outside loop $\mathcal{E} - V_{R_2} = 0$

Both R_1 & R_2 have $V_{R_1} = V_{R_2} = \mathcal{E}$!

Voltage across parallel elements is <u>equal</u>: connected at both ends by equipotentials

What about current?

Principle #2 says current branches

NOW ADD I's TO FIG

current in $I_{batt} = I_1 + I_2$ current out

What is total I_{batt}? use $I_1 R_1 = \mathcal{E}$ $\Rightarrow$ $I_1 = \frac{\mathcal{E}}{R_1}$

$I_2 R_2 = \mathcal{E}$ $\quad I_2 = \frac{\mathcal{E}}{R_2}$

$$\Rightarrow I_{batt} = \mathcal{E}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Recall $I_{batt} = \frac{\mathcal{E}}{R_{eq}}$ says $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$ for two parallel resistors

R_{eq} is <u>less</u> than R_1 or R_2!

As add resistors in parallel, I_{batt} <u>increases</u>: more pathways for current to follow → more current }

(like increasing cross-sect area of conductor connected across battery)

~~So~~ $I_{batt} > I_1$ or I_2

How do I_1 and I_2 compare in || ckt?

Ratio of currents:

$$\frac{I_1}{I_2} = \left(\frac{\mathcal{E}}{R_1}\right)\frac{1}{\left(\frac{\mathcal{E}}{R_2}\right)} = \frac{R_2}{R_1}$$

so if $R_2 = 2R_1$ then $I_1 = 2I_2$

Lower R → greater I

12:10 "Current divider"

Increasing I_{batt} means battery puts out more energy overall:

$$P = I^2 R = I(IR)$$

so in this case total energy put out by battery is
$$P = I_{batt}^2 R_{eq} = I_{batt} \underbrace{(I_{batt} R_{eq})}_{\mathcal{E}}$$

So increasing I_{batt} with same $\mathcal{E}$ $\rightarrow$ more power

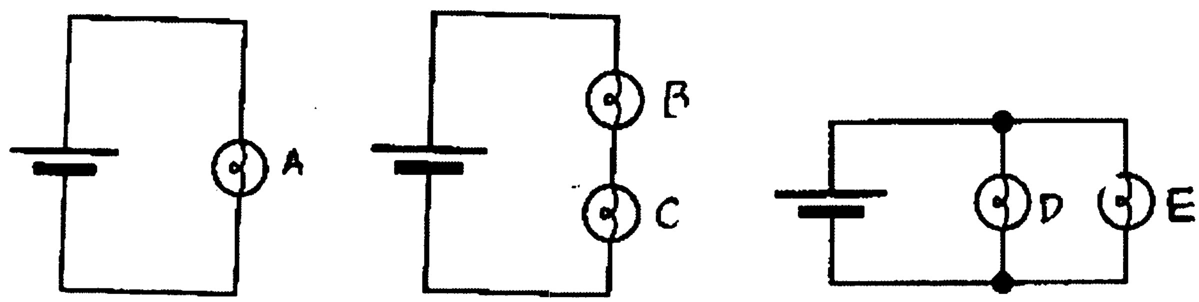
How consistent with battery giving same energy?

battery $\mathcal{E}$ = energy/charge given to <u>one charge</u>
each charge that moves through battery gains energy $q\mathcal{E}$

More charge moves through with greater I_{batt}

Assume all batteries are identical, all bulbs are identical, and the resistance of the connecting wires and internal resistance in the batteries can be ignored. Rank the brightness of the bulbs.

Brightness $\propto$ power $= I^2R$



1. A > D = E > B= C
2. D = E > A > B = C
3. D = E > A = B = C
4. A = D = E > B = C
5. A = B = C > D = E
6. Other

$V_A = V_D = V_E = \mathcal{E}$

R's all same

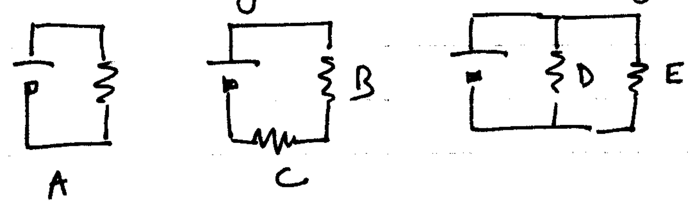
$\rightarrow I_A = I_D = I_E$

brightnesses all equal

$V_B + V_C = \mathcal{E}$

so I_B, I_C less than I_A

12:15 $\boxed{\text{CT}}$ Bulb ranking · remember brightness $\propto P = I^2 R$



Demo:

$P_A = P_D = P_E > P_B = P_C$

$V_A = V_D = V_E = \mathcal{E} \Rightarrow I_A = I_D = I_E$ b/c $I = \frac{V}{R}$ and R all same

Always (a) Ask what the question is — basically ranking currents (~~voltages~~ could think about V also)
(b) Return to the loop rule to begin with)

In this case $\underline{\underline{I_{batt}}}$ increases for the circuit w/ D & E —

the circuit as a whole draws more current, but not indiv bulbs

Short circuit: conducting path with $R=0$ in parallel to a resistor

skipped for after break

$\boxed{\text{CT}}$ bulb goes out: all current goes through short
two ways to think about it

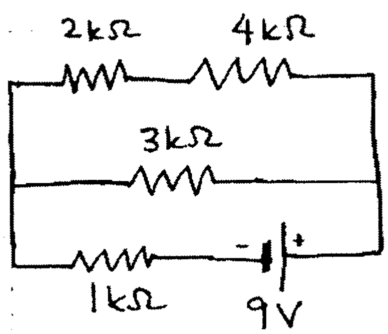
$\boxed{\text{current ratio:}}$ $\frac{I_1}{I_2} = \frac{R_2}{R_1}$ if $R_2 = 0$ then $I_1 = 0$

short is an equipotential — must have $V_{R_1} = 0$,
only possible if $I_1 = 0$ as $R_1 \neq 0$

Strategies for circuits with both series and parallel resistors:

1. If there is just one battery in the circuit, find the equivalent resistance of the circuit, and from it find the current in the battery. Then use that current and the principles of circuit analysis (loop rule, junction rule, Ohm's Law) to work your way to whatever the problem asks for.
2. If there are multiple batteries, must use loop and junction rules and write enough equations to solve for the quantity requested in the problem. Choose the loops that involve the quantity of interest.

In the circuit below, find the current in the 1 kΩ resistor, and find the potential difference across the 3 kΩ resistor.



Strategy: As this circuit has just one battery, begin by finding the equivalent resistance and using that to find the current in the battery. Then use that current to find the desired current. Then divide that current between the two upper branches of the circuit to find the desired potential difference.

Steps to follow:

1. Find the equivalent resistance of the branch with 2 kΩ and 4 kΩ.
2. Combine this branch with the 3 kΩ branch to find the equivalent resistance of both branches combined. At this point it may be helpful to draw the new equivalent circuit.

3. Combine all remaining resistances to get the equivalent resistance of the entire circuit, and use this to find the current in the battery: which is the same as the current in the 1 kΩ resistor!

4. Now return to the original circuit. Find the current in the branch with 3 kΩ by considering how the current splits between that branch and the branch with 2 kΩ and 4 kΩ, and use that current to find the potential difference across the resistor.

Strategies for circuits with both series and parallel resistors:

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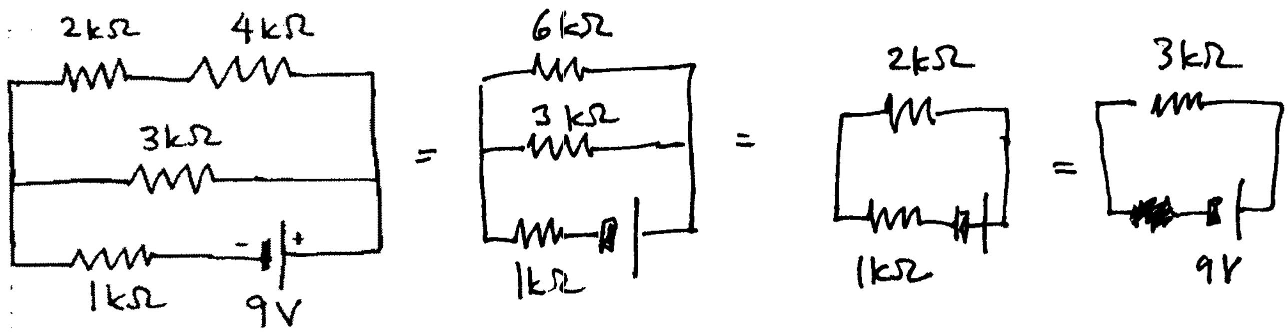


Strategy: As this circuit has just one battery, begin by finding the equivalent resistance and using that to find the current in the battery. Then use that current to find the desired current. Then divide that current between the two upper branches of the circuit to find the desired potential difference.

Steps to follow:

1. Find the equivalent resistance of the branch with 2 kΩ and 4 kΩ. $R_{eq} = 2\,k\Omega + 4\,k\Omega = 6\,k\Omega$
2. Combine this branch with the 3 kΩ branch to find the equivalent resistance of both branches combined. At this point it may be helpful to draw the new equivalent circuit.

$$\frac{1}{R_{eq}} = \frac{1}{3k\Omega} + \frac{1}{6k\Omega} \Rightarrow R_{eq} = \frac{(6\,k\Omega)(3\,k\Omega)}{(6+3\,k\Omega)} = 2\,k\Omega$$

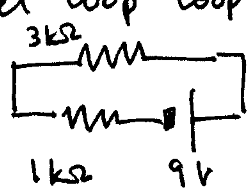
3. Combine all remaining resistances to get the equivalent resistance of the entire circuit, and use this to find the current in the battery: which is the same as the current in the 1 kΩ resistor!

$$I_{batt} = \frac{\mathcal{E}}{R_{eq}} = \frac{9\,V}{3\,k\Omega} = 3\,mA$$

4. Now return to the original circuit. [Find the current in the branch with 3 kΩ by considering how the current splits between that branch and the branch with 2 kΩ and 4 kΩ, and use that current to find the potential difference across the resistor.] Or: apply loop rule to lower loop.

Current in 1kΩ = current in battery = 3 mA !

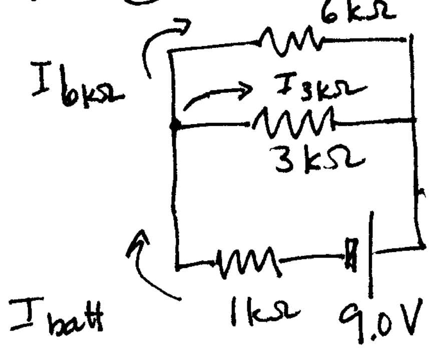
Lower loop loop rule:



$$\mathcal{E} - V_{3k\Omega} - V_{1k\Omega} = 0$$
$$\mathcal{E} - V_{3k\Omega} - I_{1k\Omega} R_{1k\Omega} = 0$$
$$9.0\,V - V_{3k\Omega} - (3\times10^{-3}A)(1\times10^{3}\Omega) = 0$$
$$\Rightarrow V_{3k\Omega} = 6.0\,V$$

If instead find current in 3kΩ branch:

look @ ckt after 1st simplification



$I_{batt} = I_{3k\Omega} + I_{6k\Omega}$ is the conservation of charge rule ("junction rule")

We have $\dfrac{I_1}{I_2} = \dfrac{R_2}{R_1}$ ~~so because~~ from requiring the voltage across both parallel branches to be equal

which becomes

$$\frac{I_{3k\Omega}}{I_{6k\Omega}} = \frac{6\,k\Omega}{3\,k\Omega} = 2$$

so $I_{3k\Omega} = 2\,I_{6k\Omega}$

We substitute this into the junction rule:

$$I_{batt} = 2\,I_{6k\Omega} + I_{6k\Omega} = 3\,I_{6k\Omega}$$

$$\Rightarrow \quad I_{6k\Omega} = \frac{1}{3} I_{batt} = \frac{3\,mA}{3} = 1\,mA$$

and $I_{3k\Omega} = 2\ mA$

Consistency check: $V_{3k\Omega} = I_{3k\Omega}(3\,k\Omega) = (2\,mA)(3\,k\Omega)$
$= 6\ V$ $\boxed{\checkmark}$