Announcements 4/1/10

Midterm 2 information handout has schedule of SA sessions and review for Saturday's exam. (April Fool — yes, the exam is Thursday 4/8 :-))

Friday afternoon office hours: 1:30 – 3 (no junior lab)

Graded homework and self-tests available in lab, then outside my office after today's lab

Key ideas from last time

Field of a loop of current on its axis:

$$B(x,0,0) = \frac{\mu_0 I a^2}{2\left(x^2 + a^2\right)^{3/2}} \qquad \text{factor of 2 in denominator!}$$

x = distance along axis
a = loop radius

For $x >> a$, $B(x,0,0) \approx \dfrac{\mu_0 I a^2}{2x^3}$

Current loop is a *dipole*: $B \propto 1/x^3$, overall field has dipole shape
Define magnetic dipole moment:

$\vec{\mu} = I\pi a^2$ in direction of $\vec{B}$ on axis   factor of π!

allows us to write $B(x,0,0) \approx \dfrac{\mu_0 \mu}{2\pi x^3}$ factor of 2π!

BTW: Earth's magnetic field is also a dipole field and is thought to come from a loop of current *inside* Earth— but explaining *why* there is a current will have to wait for next week!

Torque on dipoles in external magnetic field (NOT B_{dipole}):

rotates dipole so that $\vec{\mu}$ aligns with (parallel to) $\vec{B}$

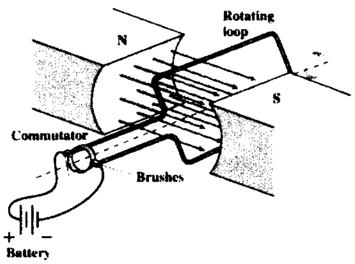
$\vec{\tau} = \vec{\mu} \times \vec{B}$

magnitude: $\tau = \mu_\perp B = \mu B \sin\theta$

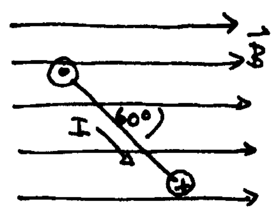
direction:

Problems for class 4/1/10

Suppose the electric motor in a Prius hybrid car has a rectangular coil 10 cm x 20 cm with 1000 turns. The motor operates at 500 V and has a 50 kW power output. The magnetic field produced by the magnets in the motor is 0.050 T. What is the torque on the motor coil when it is in the orientation shown?

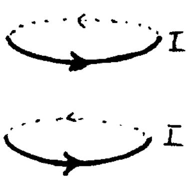


3D view



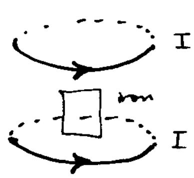
side view

What is the direction of the force on the lower current loop due to the upper current loop?



Suppose each magnetic particle in a bacterium is a sphere 50 nm in diameter made of material that has a magnetic moment per volume of $M = 5 \times 10^5$ A/m. What is the energy required to rotate one such particle from having its dipole moment parallel to the Earth's magnetic field to antiparallel to the Earth's magnetic field? (Assume the strength of the Earth's magnetic field is 60 μT.) How does this compare to the thermal energy $k_B T$ at room temperature, which is 4.1×10^{-21} J? Why do you suppose bacteria typically have 10 to 20 of these particles?

A piece of iron is magnetized by placing it between two current loops as shown. What is the direction of the magnetic moment of the iron?



J.J.
4/1/2010

(1) Review RH rules
(2) $\vec{\mu}$ of any current loop: $\vec{\mu} = NI\vec{A}_{loop}$
(3) Torques on magnetic dipoles: electric motor
(4) Energy of dipoles in field:
magnetic sensing
NMR if time
(5) Magnets as dipoles

Right-hand rules

(1) Direction of cross product: $\vec{A} \times \vec{B}$
use: RH fingers in direction of $\vec{A}$
hold hand so you can curl fingers to $\vec{B}$
cross product is in direction of thumb

Wolfson Fig 26.3

This applies to $\vec{F} = q\vec{v} \times \vec{B}$ (remember to allow for q possibly being negative)
and $\vec{F} = I\vec{L} \times \vec{B}$
(which comes from)
and $\vec{\tau} = \vec{\mu} \times \vec{B}$

(2) Direction of $\vec{B}$ of current:
$\vec{B}$ forms loops around I
point thumb along current; RH fingers curl in direction of $\vec{B}$

Wolfson Fig 26.13

(3) (Not really a new rule but a shortcut)
$\vec{\mu}$ of current loop is defined to point in direction of $\vec{B}$ on axis of current loop — can find this by how $\vec{B}$ loops around current
Also: if you curl RH fingers in direction of current, thumb points in direction of $\vec{B}$ on axis & of $\vec{\mu}$

<u>Magnetic dipole moment</u> of any current loop or coil

For single circular loop we defined

$\vec{\mu} = I\pi a^2$ in direction of $\vec{B}$ on axis

Can write in terms of loop area:

$$\vec{\mu} = I\vec{A}_{Loop}$$

$\vec{A}_{loop}$: $\begin{cases} \text{magnitude} = \text{area of loop} \\ \text{direction} = \text{same direction as } \vec{B} \text{ on axis} \end{cases}$

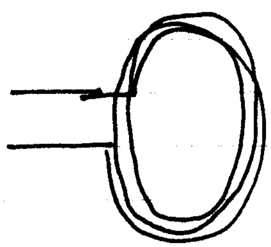
[in general: next week you'll work with a vector area in lab – vector area is always $\perp$ to the plane in which the surface exists][for magnetism we pick it to be in the same direction as $\vec{B}$ on axis]

Turns out that in general, any shape loop,

$$\vec{\mu} = I\vec{A}_{Loop}$$

Can make a coil with multiple "turns": wrap wire around multiple times

Example: 3 turn coil



Three turns $\rightarrow$ coil has 3x the $\vec{B}$ of just one loop

in external $\vec{B}$, coil feels 3x the torque of just one loop

$$\Rightarrow \vec{\mu} = NI\vec{A}_{Loop}$$

where N = # of turns

Sample problem: electric motor

How do motors work:

motor needs to take electric energy from a current and use it to turn something – i.e. axle of car

pass current through motor coil – it feels a torque due to magnets mounted around it

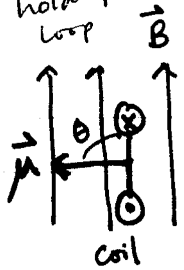
Start with $\vec{\mu}_{coil} \perp$ to $\vec{B}$: it feels torque

$$\vec{\tau} = \vec{\mu}_{coil} \times \vec{B} = \mu_{coil} B \sin\theta \quad \text{in direction from cross prod}$$

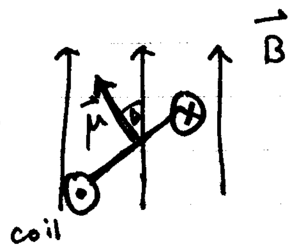
Direction of torque: if you use the cross product rule get a direction along the axis of rotation

I think it makes more sense for our purposes to indicate the way the coil rotates: for example, viewed like this the coil rotates clockwise

Draw on board hold up cardb loop



$\Rightarrow$



Magnitude of torque is

$$\tau = \mu_\perp B = \mu B \sin\theta$$

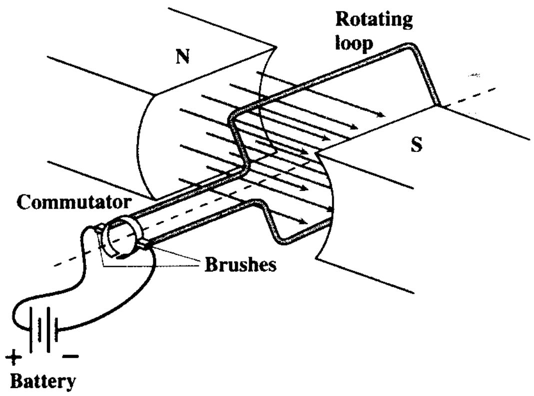
where θ = angle between μ and B

Motors use a special trick so that torque keeps them turning in same direction all the time instead of reversing: a way to switch current direction

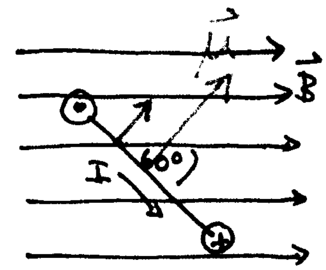
Ask: direction of torque once just past parallel?

Easiest to see on a 3D object – we'll look at this in lab in 2 weeks

Suppose the electric motor in a Prius has a rectangular coil 10 cm x 20 cm with 1000 turns. The motor operates at 500 V and has a 50 kW power output. The magnetic field produced by the magnets in the motor is 0.050 T. What is the torque on the motor coil when it is in the orientation shown?



3D view



τ causes CW rotation

side view

principle: $\vec{\tau} = \vec{\mu} \times \vec{B}$ magnitude $\tau = \mu B \sin\theta$

Need $\mu, \theta \Rightarrow \theta = 30°$ between $\vec{\mu}$ and $\vec{B}$

$\mu = N I A_{loop}$ need I

$\text{Power} = IV \Rightarrow I = \dfrac{\text{power}}{V}$

$\mu = (1000)(100\,A)(2.0 \times 10^{-2}\,m^2) = 2000\,A\,m^2$

$\Rightarrow \tau = 50\ N \cdot m$

Direction: causes loop to rotate CW in side view

Prius problem

Goal: Need to find the torque on the motor

Principle: Current loop experiences ~~torque~~ $\vec{\tau} = \vec{\mu} \times \vec{B}$ in magnetic field

Need to find $\vec{\mu}$, given loop geometry and motor power and voltage

$\vec{\mu}_{loop} = N I \vec{A}_{loop}$ need current

To find current: recall power = IV in general

so $I_{motor} = \dfrac{P_{motor}}{V_{motor}} = \dfrac{50 \times 10^3\,W}{500\,V} = 100\,A$ (a lot!)

Area of loop: $= \cancel{\pi\left(\frac{0.20\,m}{2}\right)^2} = 2.0 \times 10^{-2}\,m^2$

10 cm x 20 cm

$\Rightarrow \mu = N I A_{loop} = (1000 \text{ turns})(100\,A)(2.0 \times 10^{-2}\,m^2) = 2000\,A \cdot m^2$

Now use $\tau = \mu B \sin\theta$ with angle shown as 30° — this is the angle between $\vec{\mu}$ and $\vec{B}$ so it's the right one

$\Rightarrow \tau = (2000\,A\,m^2)(50 \times 10^{-3}\,T) \sin 30°$

$\tau = 50\,N \cdot m$

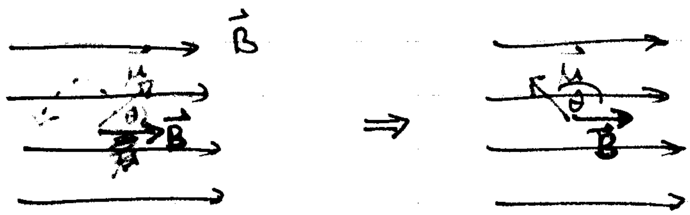
What if you want more torque? Increase N (harder to increase I or B)

Energy of dipoles in $\vec{B}$

To move dipole away from $\vec{\mu} \parallel \vec{B}$ requires pushing on it – must do work and therefore ~~store~~ increase potential energy

(Draw $\vec{B}$ on board, show arrow rotating)

show at two arbitrary angles $\theta_{initial}$, θ_{final}



To find work required to go from $\theta_{initial}$ to θ_{final}:

$$W = \int_{\theta_{initial}}^{\theta_{final}} \tau \, d\theta = \int_{\theta_{initial}}^{\theta_{final}} \mu B \sin\theta \, d\theta = -\mu B \cos\theta \Big|_{\theta_{initial}}^{\theta_{final}}$$

$$\text{so} \quad \Delta U = -\mu B (\cos\theta_{final} - \cos\theta_{initial})$$

~~Book chooses~~

Most relevant energy difference: from parallel (0°) to anti (180°)

$$\Delta U = -\mu B (\cos 180° - \cos 0°) = 2\mu B$$

anti parallel is <u>higher</u> in energy by $2\mu B$

this much energy is required to reverse $\vec{\mu}$ from its lowest energy orientation

[Your book chooses reference to be $\theta_{initial} = 90°$ and then $U_{\theta_{final}} = -\mu B \cos\theta_{final} = -\vec{\mu} \cdot \vec{B}$] in final position

— Exam material stops here —

Magnetic materials: How does all this relate to what we colloquially call "magnets"?

What you may have used or played around with as a kid: a chunk of material that picks up paper clips (but not copper pennies) and sticks to your refrigerator (but not aluminum foil)

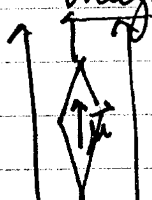
Mazur Fig 31.3

If you have two of them, you find that a magnet has two ends, which we call "north" and "south" poles, and that like poles repel while unlike poles attract — very strongly, in fact!

Where does the name north/south come from? Compass is a sliver of magnet ~~ets~~ set on a ~~bearing~~ pivot so it can rotate — observed long ago that it turns so that its "north" end goes toward magnetic north.

Mazur Fig 31.2

In fact this rotation happens b/c the compass needle is a magnetic dipole w/ its $\vec{\mu}$ from S to N $\therefore$ feels a torque from Earth's $\vec{B}$, rotates so it aligns parallel to $\vec{B}_{earth}$ (which points from ~~t~~ the south to the north).

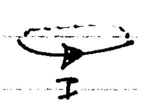

$\vec{\mu}$

Any chunk of magnet turns out to be a magnetic dipole — its field is the same form as a current loop (Fig 32.8)

Figs 31.7-8

Might think N pole and S pole are like ⊕ and ⊖

$\boxed{\text{BUT}}$ if you cut a magnet in half, get two new dipoles each with S and N — all the way to the atomic level!

So can think interchangeably about magnets, current loops, & dipoles: $\boxed{\begin{matrix} N \\ S \end{matrix}} = \uparrow\vec{\mu} =$  I

Magnet physics: Mazur, *Physics: Principles and Practice* (Addison-Wesley, to be published).

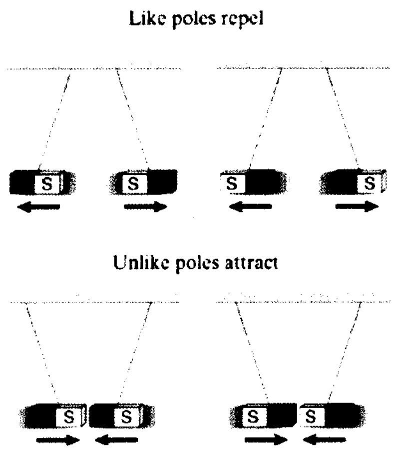


Figure 31.3 Interactions between magnetic poles.

Magnets have two distinct "poles"
"north" and "south"

like poles repel
opposite poles attract

Poles are named this way
because N end of magnet
turns toward geomagnetic
north (and likewise S)

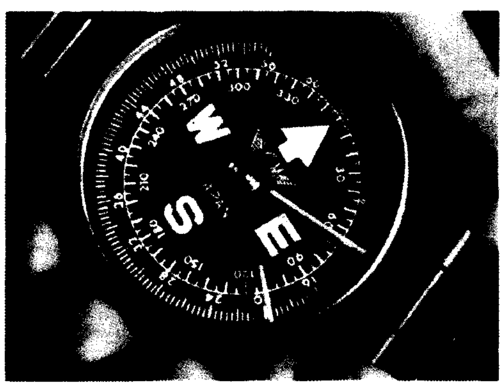
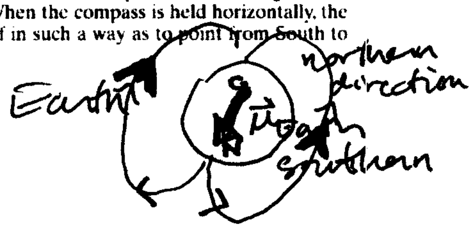
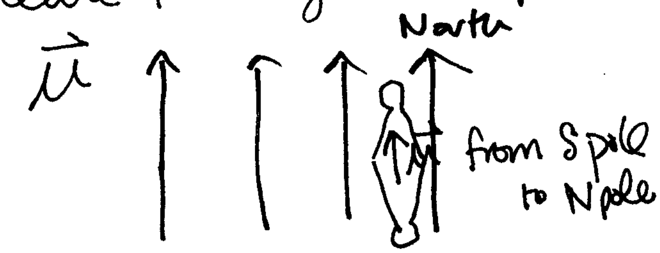


Figure 31.2 The needle of a compass is a small magnet that is free to rotate. When the compass is held horizontally, the needle aligns itself in such a way as to point from South to North.



Needle is magnetic dipole



torque aligns $\vec{\mu}_{compass}$ to $\vec{B}_{Earth}$

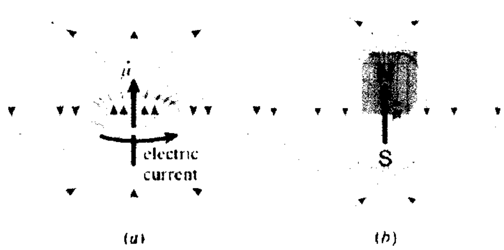
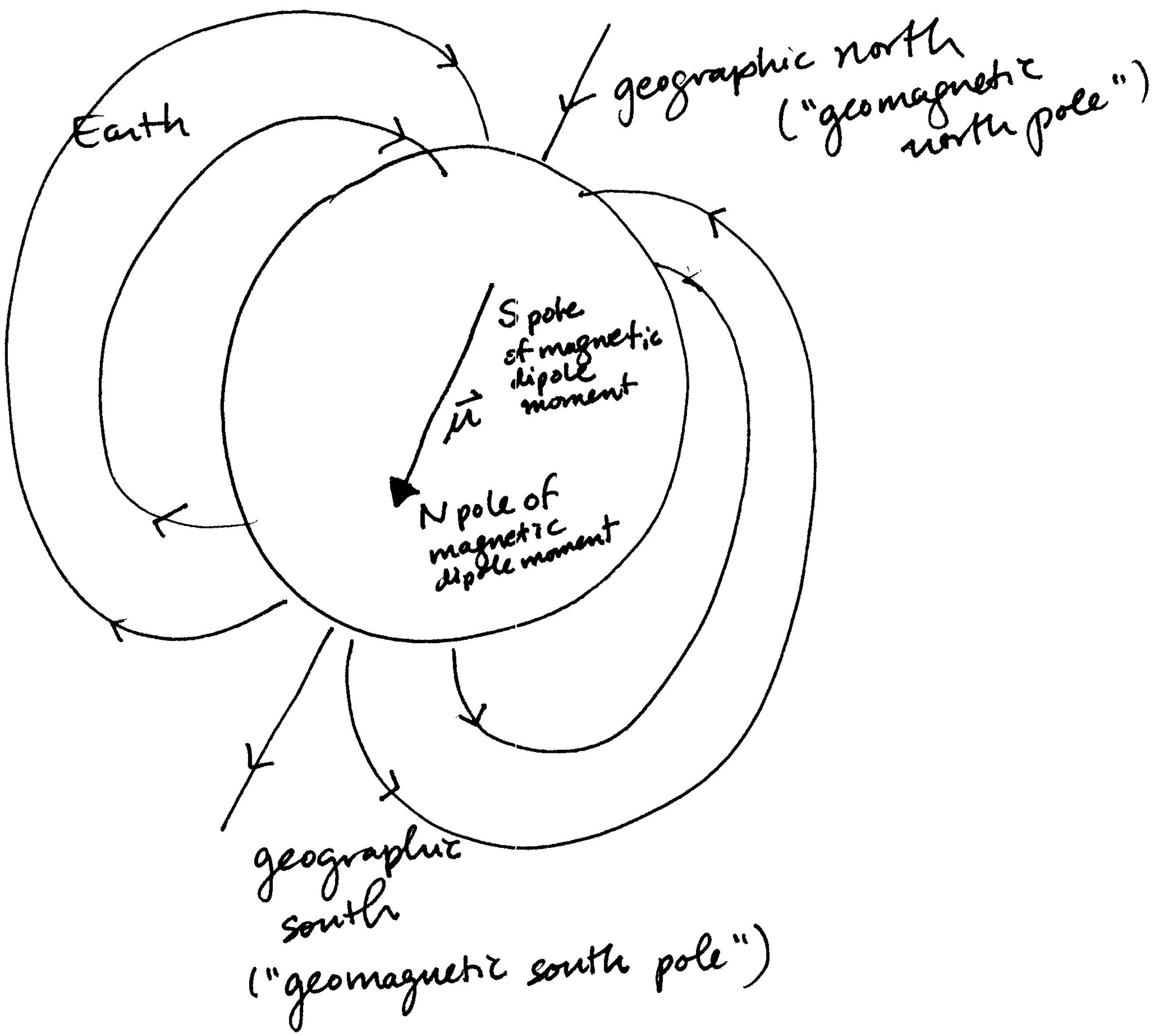


Figure 32.8 The magnetic dipole moment vector $\vec{\mu}$ points along the axis of a (*a*) current loop and (*b*) bar magnet, in the same direction as the magnetic field.

"magnets" are dipoles

Clearer picture of Earth's field and dipole moment



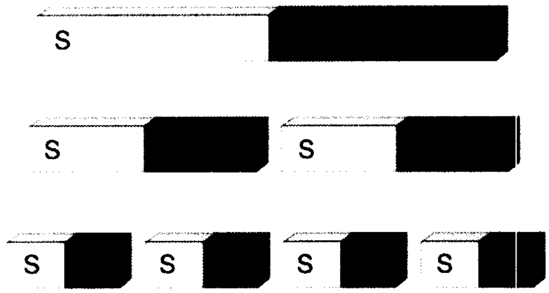


Figure 31.7 When a magnet is cut in two, each piece retains both a N and S pole.

Does this mean

$N = \oplus$

$S = \ominus$?

No – if break in half → more dipoles

B/C magnetism comes from loop of current

= "Spin" of e^-, nuclei

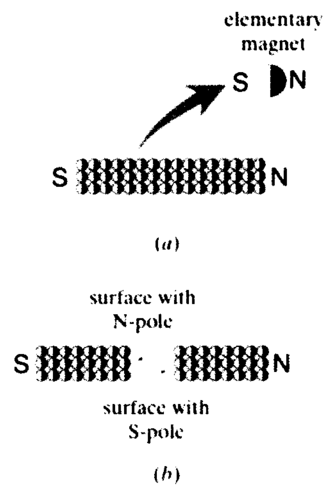


Figure 31.8 (*a*) In a magnetized piece of material, all the elementary magnets are aligned. (*b*) Cutting the piece in two exposes two new poles.

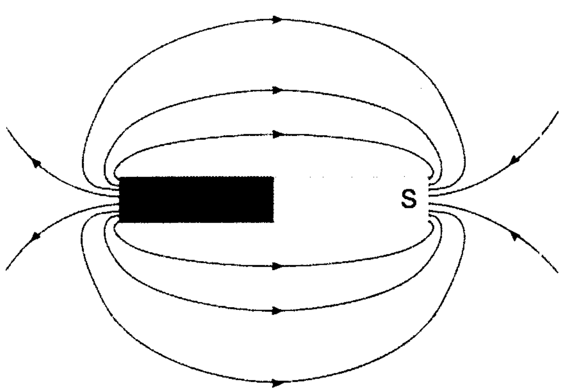
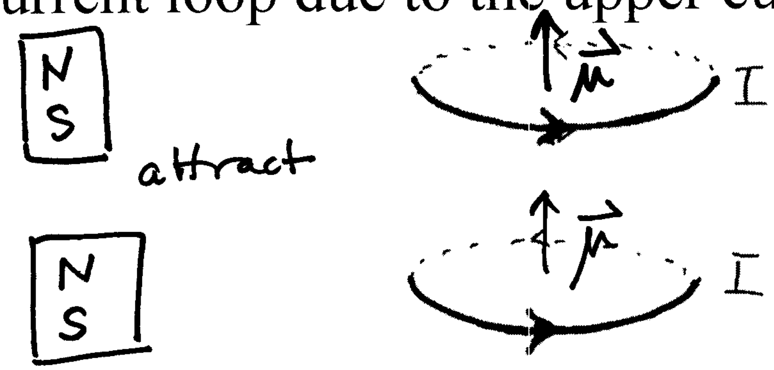


Figure 31.13 Magnetic field line pattern surrounding a bar magnet.

What is the direction of the force on the lower current loop due to the upper current loop?



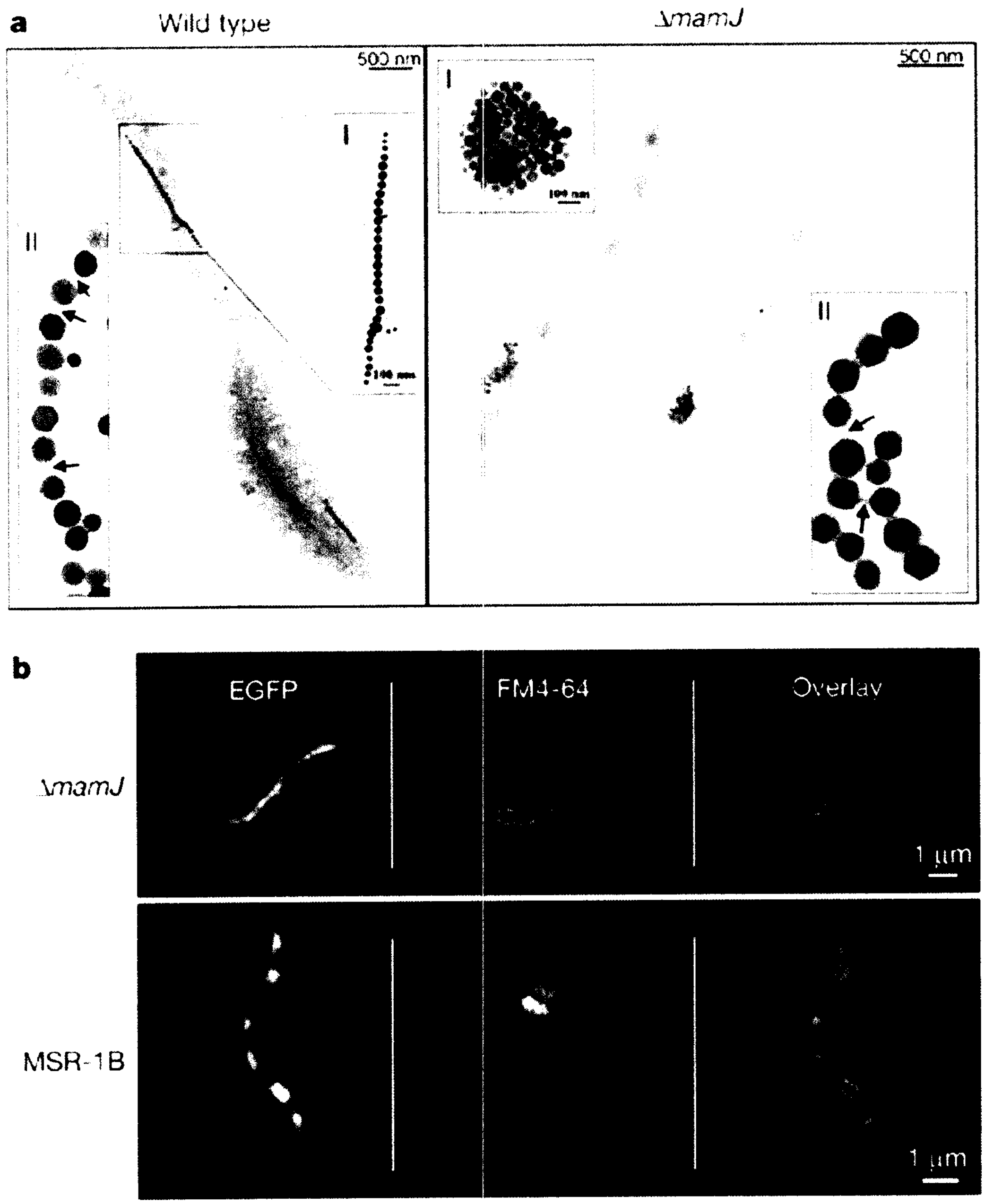
Can use $I\vec{L}\times\vec{B}$ also

$\vec{B}_{top}$ into page on lower loop at front

$\rightarrow \vec{F}$ up

1. Left
2. Right
3. Up
4. Down
5. Into page
6. Out of page
7. There is zero net force.

From Nature 440, 110–114 (2 March 2006), "An acidic protein aligns magnetosomes along a filamentous structure in magnetotactic bacteria," by André Scheffel, Manuela Gruska, Damien Faivre, Alexandros Linaroudis, Jürgen M. Plitzko and Dirk Schüler



$\times 10^{-9}$ m

Suppose each magnetic particle in a bacterium is a sphere 50 nm in diameter made of material that has a magnetic moment per volume of M = 5 x 10^5 A/m. What is the energy required to rotate one such particle from having its dipole moment parallel to the Earth's magnetic field to antiparallel to the Earth's magnetic field? (Assume the strength of the Earth's magnetic field is 60 μT.) How does this compare to the thermal energy k_BT at room temperature, which is 4.1×10^{-21} J? Why do you suppose bacteria typically have 10 to 20 of these particles?

$$\Delta U_{\text{to flip}} = 2\mu B$$

$$\mu = M\ \text{vol} = M\,\tfrac{4}{3}\pi\left(\tfrac{d}{2}\right)^3 = 3\times10^{-17}\ \text{A m}^2$$

$$B_{\text{earth}} = 60\times10^{-6}\ \text{T}$$

$$\rightarrow \Delta U_{\text{to flip}} = 3.6\times10^{-21}\ \text{J}$$

$$kT \text{ at room t} = 4.1\times10^{-21}\ \text{J}$$

$$\Delta U_{\text{to flip a chain of 10}} = 10(3.6\times10^{-21}\ \text{J}) = 3.6\times10^{-20}\ \text{J}$$

Applications:
mag sensing
NMR

(could save for after ferro/para)

Magnetic sensing/navigation

Many kinds of creatures use the torque of Earth's $\vec{B}$ on magnetic particles to orient themselves — some for migration, some for other navigational purposes

(ants & other insects, fish) (birds may be more complex)

Simplest: magnetotactic bacteria

stream/pond dwellers

anaerobic – need to find their way to the bottom of the water for food and to avoid oxygen

DIP NEEDLE DEMO

in N hemisphere, $\vec{B}$ of earth points mostly down — so can follow ~~Earth's~~ field to go down

(Why not use gravity? Not strong enough — bacteria are not significantly denser than the water surrounding them)

Bacteria contain a chain of 10-20 tiny magnetic particles each ~50 nm in diameter

$\mu \cong 3\times10^{-17}\ A\cdot m^2$ for each

$\mu = M\cdot vol = M\,\frac{4}{3}\pi\left(\frac{d}{2}\right)^3$

Is torque strong enough to keep bacteria following $\vec{B}_{earth}$?

Do energy here

Difference in energy between aligned & antialigned:

$$\Delta U = 2\mu B = 2(3\times10^{-17} A\cdot m^2)(6\times10^{-6}T) = 3.6\times10^{-21} J$$

Ten particles requires 10x energy: $\Delta U = 3.6\times10^{-20}$ J

Room temperature: thermal energy is 4.1×10^{-21} J — so 10 particles will keep bacteria pointing down — not enough thermal energy to push them around.

Stopped here 12:30

CT?

(How do they know this is right? Take bacteria to S hemisphere where $\vec{B}$ points up — bacteria go to top & die.)