Announcements 3/30/10

SA + review session info

Midterm information sheet online will update details of what's covered
PS 9 covers remaining material for midterm 2, due next Tuesday

Friday afternoon office hours this week: 1:30 – 3 (no junior lab)

Reading for Thursday: additional reading on magnets (handout & posted online)

Wolfson reading: read after class in all cases

Thurs 3/25: we're using the results of Exs. 26.4 and 26.5, which are Eqs. 26.9 and 26.10. Also read "Magnetic force between conductors" (p. 447)

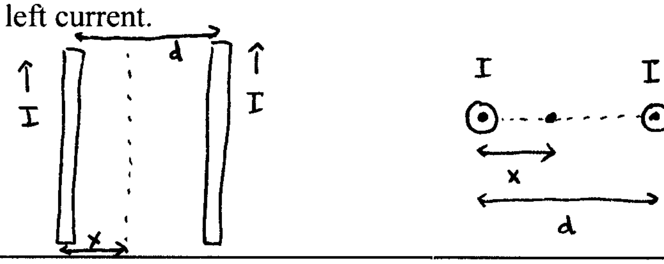
Today 3/30: 26.6 but ignore "Gauss's Law for magnetism"

Thursday 4/1: 26.7, and "Solenoids" 454-456, except ignore the Ampere's Law argument for it. Use our class discussions as a guide to what is important in 26.7.

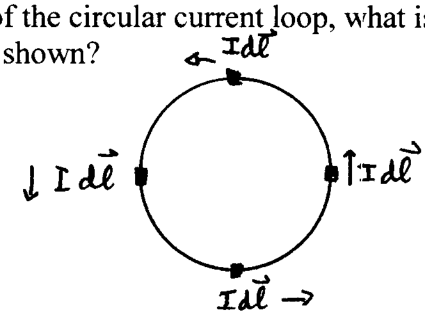
Volunteers needed to help get board really clean before class – if you're early anyway

Problems for class, 3/30/10

Find the combined magnetic field (magnitude and direction) of two parallel currents a distance x from the left current.

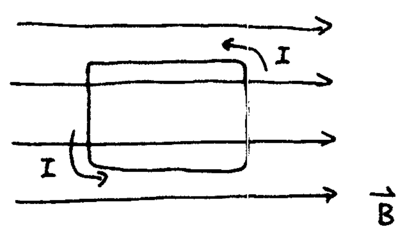


At the center of the circular current loop, what is the direction of the magnetic field due to the four segments shown?

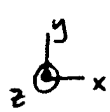


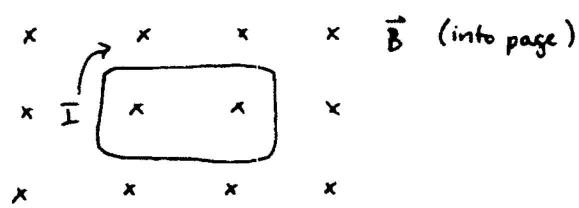
A circular wire loop of radius 15 cm and negligible thickness carries a 2.0 A current clockwise as viewed from above. Use suitable approximations to find the magnetic field of this loop (a) in the plane of the loop, 1.0 mm outside the loop, (b) on the loop axis, 60 cm above the center of the loop.

A current loop is placed in a magnetic field as shown below. Which way does the loop rotate?



A current loop is placed in a magnetic field as shown below. Which way does the loop rotate?





Key ideas from last time

Obtained magnetic force on current by adding up forces on individual moving charges

$\vec{F} = I\vec{L} \times \vec{B}$

force by uniform $\vec{B}$ on straight current I, length L

$\vec{L}$ points in direction of current = direction of $q\vec{v}_d$

(*opposite* to motion of negative charges!)

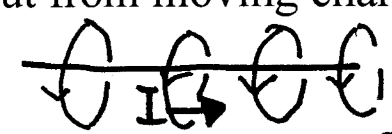
Circuits (closed loops of current):

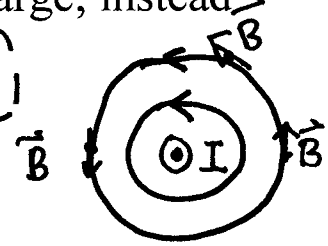
no net magnetic force on circuit from uniform $\vec{B}$

do experience a magnetic force from nonuniform $\vec{B}$

Source of magnetic field is moving charge = current

BUT field lines do not point out from moving charge; instead they loop around it





Magnetic field of long straight current:

Strength $B = \dfrac{\mu_0 I}{2\pi r}$

r = perpendicular distance from current

Direction: field lines form right-handed loops around I

At any point $\vec{B}$ is *tangent* to the field line

J.J.

3/30/2010

Today

(1) Superposition of magnetic fields:
find $\vec{B}$ by adding up fields of individual currents
- $\vec{B}$ of two parallel wires midway between
- Biot-Savart law
- $\vec{B}$ of $\boxed{\text{circular loop of current}}$ (most important/physically relevant building block)

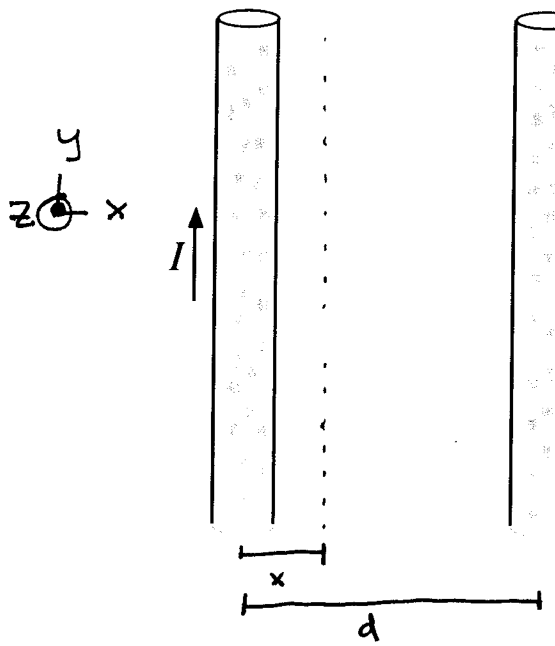
(2) Current loop = magnetic dipole
- $\vec{B}$ of loop = dipole field far away
- forces on loops: no force in uniform $\vec{B}$
force in nonuniform $\vec{B}$
- torques on loops: loop rotates to have its dipole moment parallel to $\vec{B}$

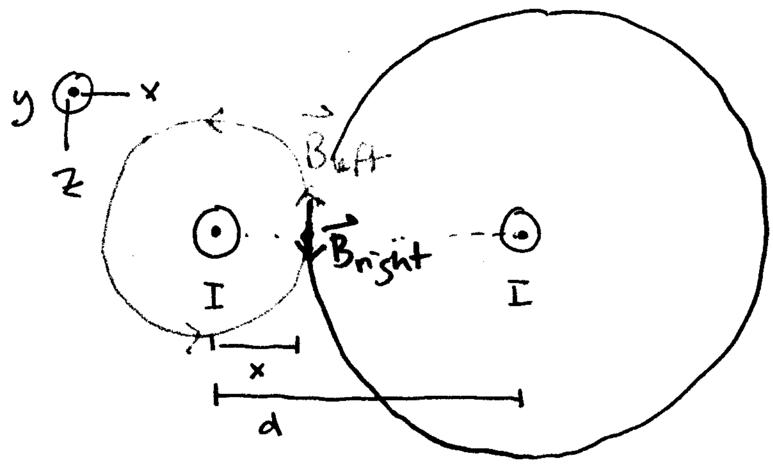
Define $\vec{\mu} = NIA$ with direction from third RH rule

(3) Magnetic materials have current loops at the atomic level due to electron spin and orbital motion!

(4) Review 3 RH rules
(1) Cross products
(2) Direction of $\vec{B}$ around long straight wire
(3) Direction of $\vec{\mu}$ of a current loop

Find the combined magnetic field (magnitude and direction) of two parallel currents a distance x from the left current.





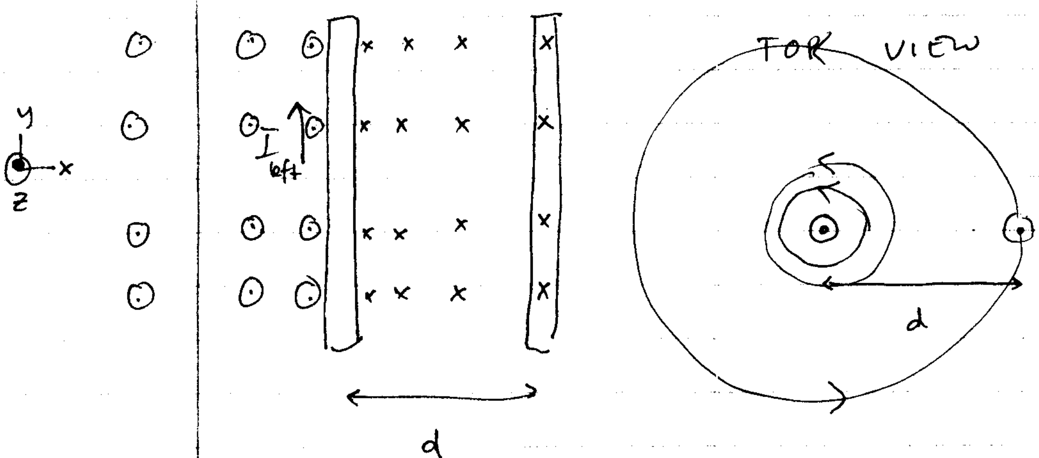
Add $\vec{B}_{left} + \vec{B}_{right}$

$$\vec{B}_{left} = \frac{\mu_0 I}{2\pi x}(-\hat{k})$$

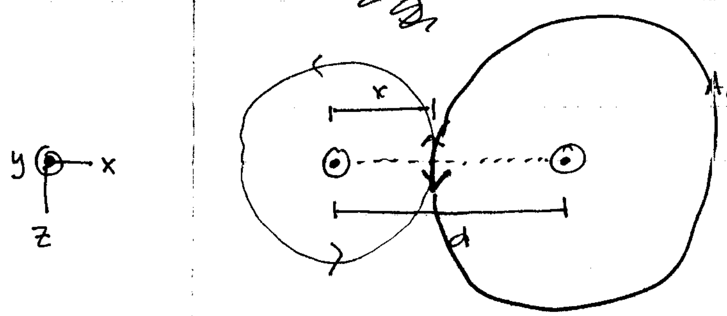
$$\vec{B}_{right} = \frac{\mu_0 I}{2\pi(d-x)}(\hat{k})$$

$$\vec{B}_{total} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{d-x} - \frac{1}{x}\right)\hat{k}$$

Last time we found $\vec{B}$ field of left wire at right wire b/c we wanted force by left on right:



Instead what if we want the total $\vec{B}$ due to both wires somewhere along the line between the wires?



Add fields together

$$\vec{B}_{left} = \frac{\mu_0 I_{left}}{2\pi x}(-\hat{k})$$

$$\vec{B}_{right} = \frac{\mu_0 I_{right}}{2\pi (d-x)}(\hat{k})$$

$$\Rightarrow \vec{B}_{total} = \frac{\mu_0}{2\pi}\left(\frac{I_{left}}{x}(-\hat{k}) + \frac{I_{right}}{d-x}\hat{k}\right)$$

If $I_{left} = I_{right} = I \Rightarrow \vec{B}_{total} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x}(-\hat{k}) + \frac{1}{d-x}(\hat{k})\right)$

If two currents are equal, at what x will $\vec{B}_{total} = 0$?

at $x = d/2$ – halfway

Expr for B assumes long wire and $r \ll L$

$\Rightarrow$ This is approximately true as long as d is small compared to length of wire

Biot-Savart Law

We are not going to use the Biot-Savart Law quantitatively - instead simply understand what it means qualitatively

- If ~~the~~ we have a current that is a loop or other complicated shape, we find its $\vec{B}$ by dividing the current into straight segments of length dl
- Each segment produces $\vec{B}$ that forms circular loops around it with magnitude $\frac{\mu_0 I}{4\pi r^2} d\vec{l}$
- Total $\vec{B}$ = sum of $\vec{B}$'s of segments
  → magnetic field strength increases w/ I increasing & at any point, the contribution is greatest from nearby segments and weaker from segments that are farther away

$\boxed{\text{For board}}$

Biot-Savart

~~Complicated shape~~

Current that is not straight: divide into straight segments

total $\vec{B}$ = ~~[illegible]~~ sum of $\vec{B}$'s of segments

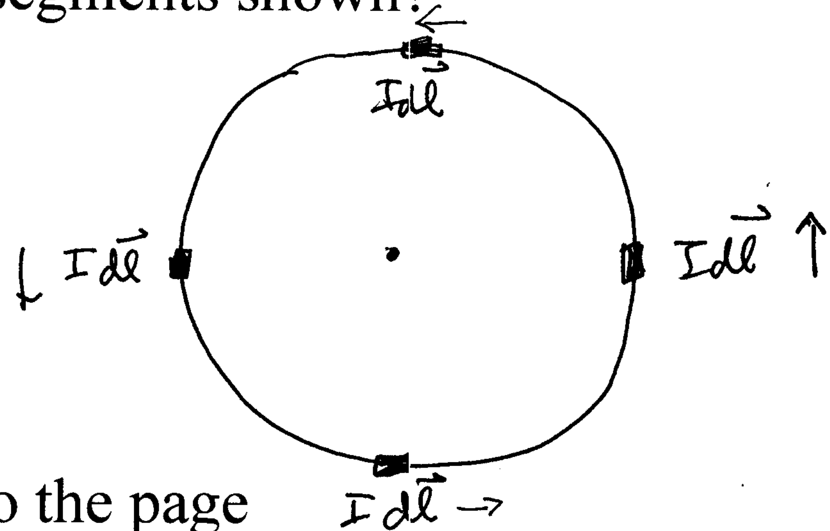
each segment has $\vec{B}$ that forms circular loops around it

$B \propto \mu_0 I$ and decreases w/ distance r

Nearby segments contribute most

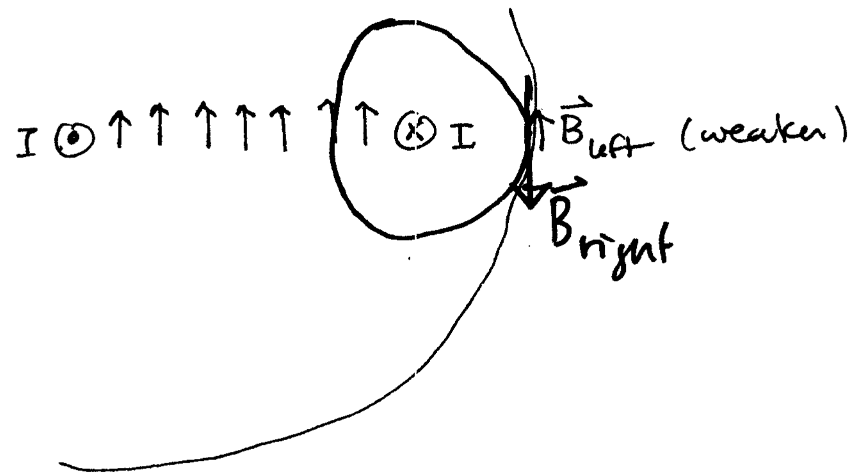
Farther segments contribute less

At the center of the circular current loop, what is the direction of the magnetic field due to the four segments shown?



1. into the page
2. out of the page R.H rule for current loops
3. to the left
4. to the right
5. The magnetic field due to those segments is zero at the center of the loop.

Look at the same loop from the side. Directly to the right of the loop, what is the direction of the magnetic field due to the two segments shown?



1. up
2. down
3. to the left
4. to the right
5. into the page
6. out of the page

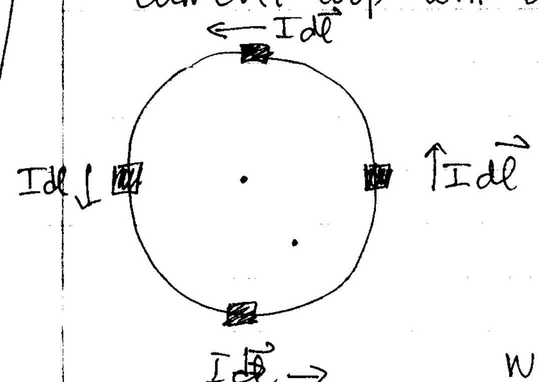
Circular loop of current

Turns out most important current arrangement is a circular loop of current ~~- basic building block of most magnets~~
~~- when produce magnetic fields by~~

- "Electromagnets" = current-carrying wires designed for the purpose of producing magnetic fields are usually either circular loops or stacks of such loops
- Fundamental basis of magnetic materials = atomic current loops in material

11.45

Think qualitatively about what field of a circular current loop will be by subdividing it into segments



$\boxed{CT}$: net $\vec{B}$ from 4 segments shown @ ctr of loop

Out of page – all four ~~wires~~ segments $\Rightarrow \vec{B}$ in that direction

What about off-center? ~~Direction is still out of page~~ ~~also all of loop $\Rightarrow$ field out of page inside, into page outside loop~~

If pick a location inside the loop, all of the current in the loop $\Rightarrow$ $\vec{B}$ pointing out of page – pick any segment

Outside loop: draw side view of loop

Use 3D props (cardboard loops)

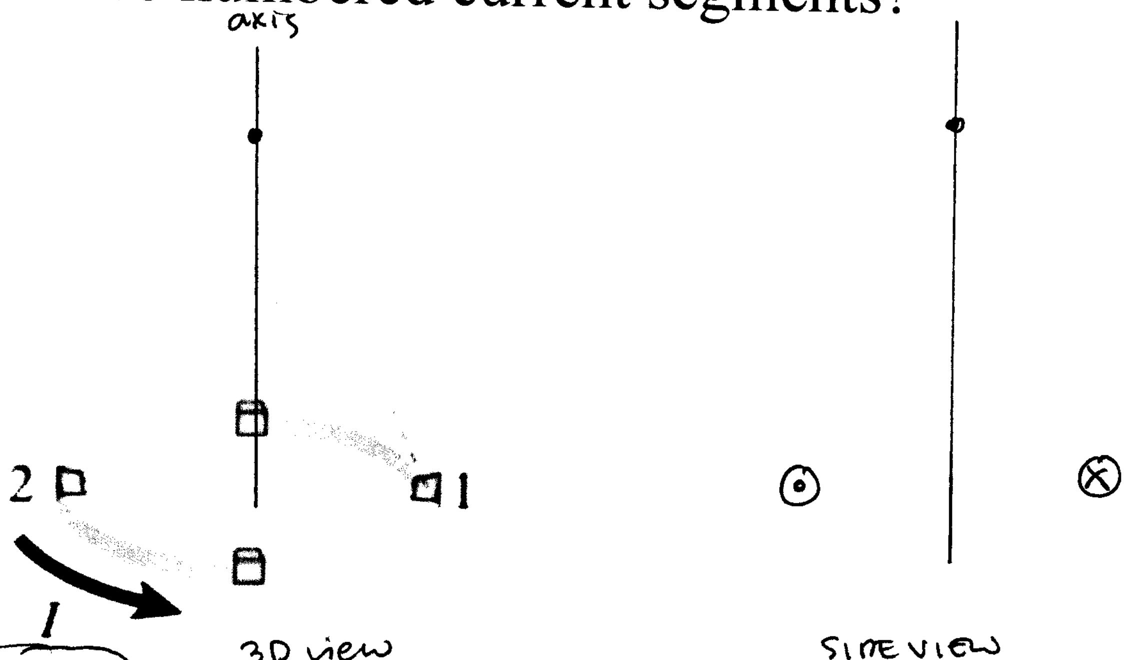
I $\qquad\qquad\qquad$ I

$\odot \uparrow \uparrow \uparrow \uparrow \uparrow \otimes \cdot$

$\boxed{CT}$ Direction of $\vec{B}_{loop}$?
Down – nearer side of loop $\rightarrow$ field

~~Show 1st pg of $\vec{B}$ handout: idea is that we're wrapping the wire into a circular loop, and adding up fields of each~~

On the axis of the circular current loop shown, what is the direction of the magnetic field due to the two numbered current segments?

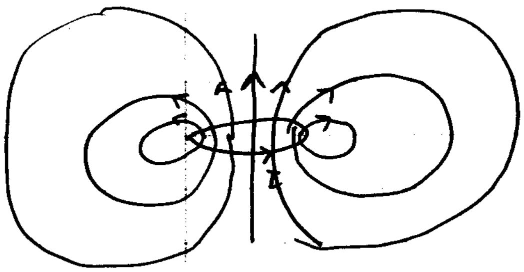


1. up
2. down
3. to the left
4. to the right
5. into the page
6. out of the page

What about on the axis higher up or lower?

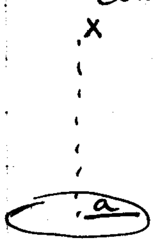
$\boxed{\text{CT}}$ Fields all add to be along the axis

Put all these together – gives us field of a current loop



Using the quantitative Biot-Savart Law gives a result for the strength of the field on the axis:

make the axis of the loop the x-axis (that's Wolfson's choice), radius of loop = a



$$B(x,0,0) = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}$$

direction of $\vec{B}$ comes from RH rule as already discussed

Limit if $x \gg a$:

$$B(x,0,0) \approx \frac{\mu_0 I a^2}{2x^3}$$

This is a dipole field!
- pattern of field looks like a dipole once you're not too close to the loop (see handout)
- mathematical form is also dipole: $B \propto 1/\text{distance}^3$

Suggests that there might be a magnetic dipole moment for a loop of current, and that basic unit of magnetic sources fields is these mag dipoles (also explains why field lines loop around)

From Mazur, *Physics: Principles and Practice*, to be published (Addison-Wesley).

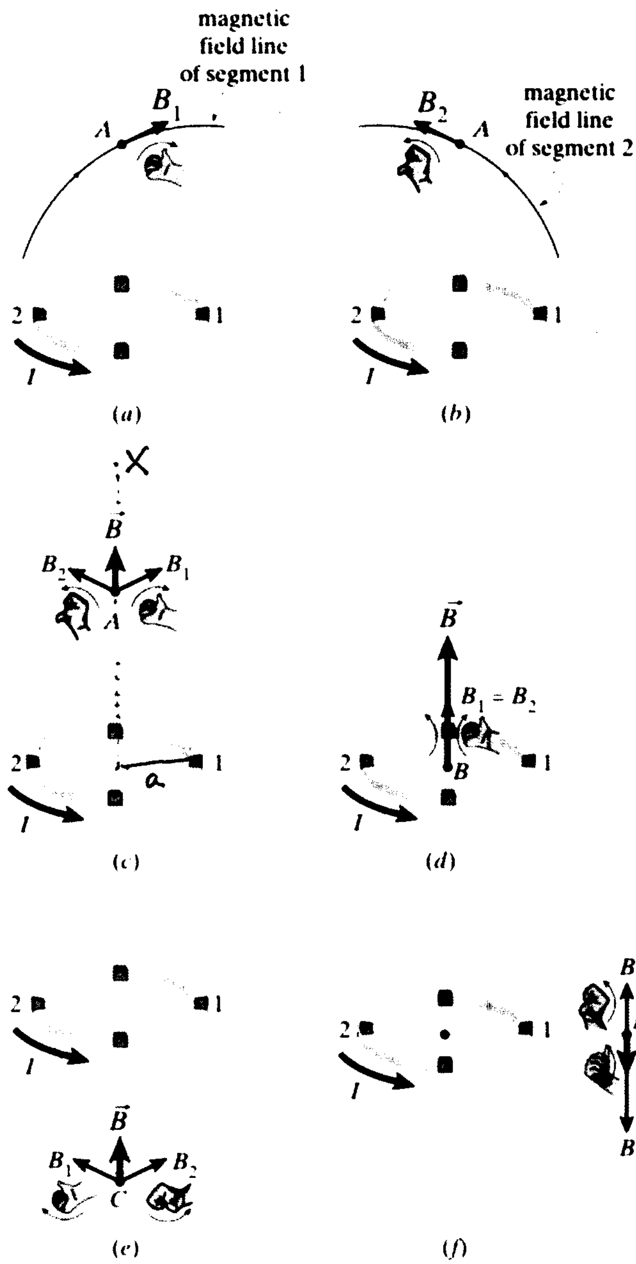


Figure 32.5 Mapping out the magnetic field of a current loop. The magnetic field contributions from (a) segments 1 and (b) 2 at A (c) add up to a vertical field. Magnetic fields at (d) at point B at the center of the ring, (e) at point C below the ring, and (f) at point D to the right of the ring. Note that in all cases the magnetic field of each segment is perpendicular to the line connecting that segment to the point at which we are determining the field.

quantitative expression for strength of B:

x = distance from loop along axis

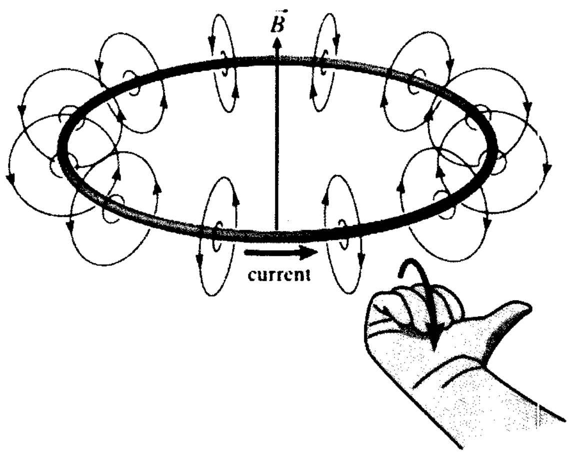
a = radius of loop

$$B = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}$$

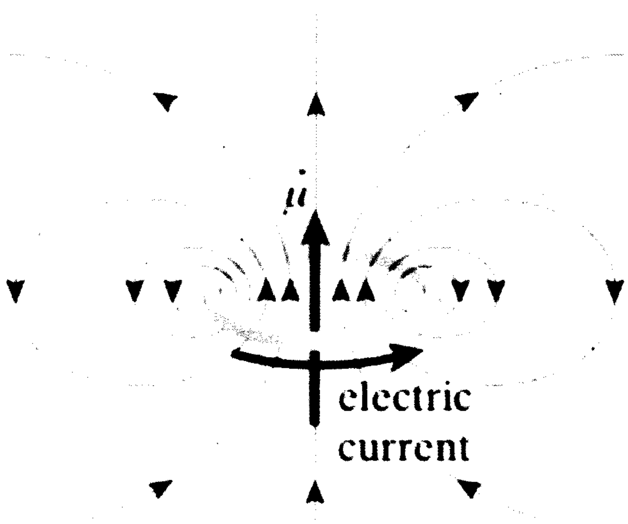
if a >> x B stronger for smaller a

~~[illegible]~~ B decreases w/increasing x

Wrapping a long current-carrying wire into a loop



produces the following field when you add together the contributions from each little segment everywhere:

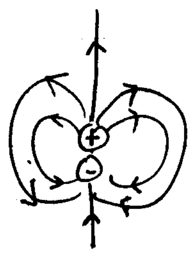


(a)

Define magnetic dipole moment to ~~[illegible]~~ give equivalent results to electric dipole moment for field, force, and torque

Look at $B(x,0,0) \cong \dfrac{\mu_0 I a^2}{2x^3} = \dfrac{\mu_0 I \pi a^2}{2\pi x^3}$ multiplying by π in both num. & denom.

Compare $E(x,0,0)$ if we put dipole axis on x-axis: (so that field looks same)



$$E = \frac{p}{4\pi\epsilon_0 x^3}$$

$I\pi a^2 = I A_{loop}$ A_{loop} = area of loop

Define magnetic dipole moment $|\vec{\mu}| \equiv$ ~~[illegible]~~ and points along axis of loop: then

$$B = \frac{\mu_0 \mu}{2\pi x^3}$$

and in both cases field points along x-axis when

How do we know what direction $\vec{\mu}$ is?

<u>$\vec{\mu}$ is parallel to $\vec{B}$ on axis</u>, so can just find direction of $\vec{B}$ from direction of current — this is direction of $\vec{\mu}$ also

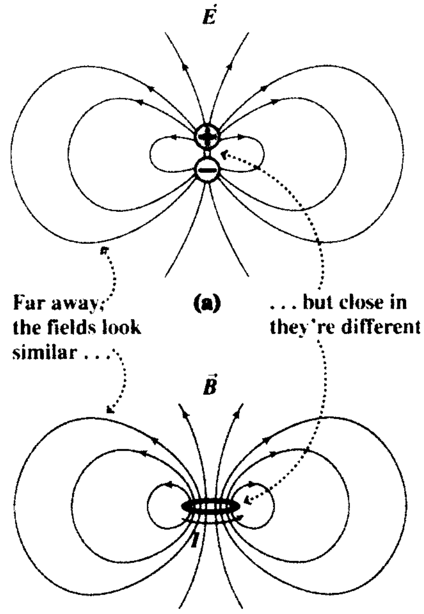
There <u>is</u> another RH rule: shown on handout

curl RH fingers along current, thumb in direction of $\vec{B}$ and $\vec{\mu}$

12:10 don't need to know this one

If multiple loops in a flat coil:

$\vec{\mu} = N I A_{loop}$ with N = # of loops



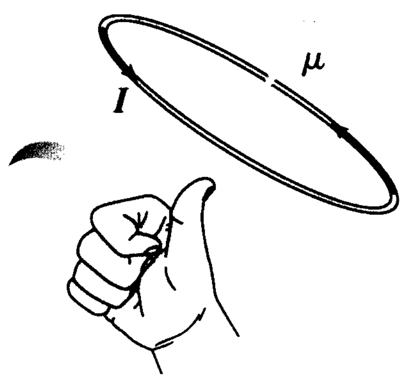
Dipole fields matter far away

→ $\vec{E}_{dipole}$ and $\vec{B}_{dipole}$ have same shape far away

Define magnetic dipole moment

$\vec{\mu} \doteq I\pi a^2$ in same direction as $\vec{B}$ on axis of current loop

$\vec{\mu} = IA_{loop}$ in direction of $\vec{B}$ on axis



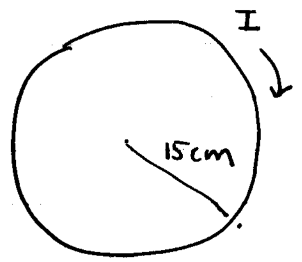
Find direction of $\vec{\mu}$ ($\vec{B}$ on axis)

by curling RH fingers along current

→ thumb in $\vec{\mu}$ direction

Sample problem with current loops:

Sketch:



ⓐ $\vec{B}$ 1 mm from loop in plane?

So close to the loop that the dominant contribution to the magnetic field is just from the nearby segment of the loop – so close that it's as if the wire was straight

$$\Rightarrow \quad B \approx \frac{\mu_0 I}{2\pi r} = \frac{(1.26\times10^{-6}\ N/A^2)(2.0\ A)}{2\pi(0.001\ m)} = 4.0\times10^{-4}\ T$$

$\perp$ dist $= 0.001\ m$

$= 400\ \mu T$

$(1\ \mu T = 10^{-6}\ T)$

This is a moderate field – ~7x Earth's field. But varies dramatically this close to wire – not a practical approach!

Direction: out of page as drawn

ⓑ $\vec{B}$ 60 cm above loop on axis?

Could use result for loop:

$$B = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}$$

with $x = 0.60\ m$ = distance along axis
$a = 0.15\ m$ = loop radius

BUT: $x^2 = 0.36$ and $a^2 = 0.02$

so can pretty reasonably use dipole form

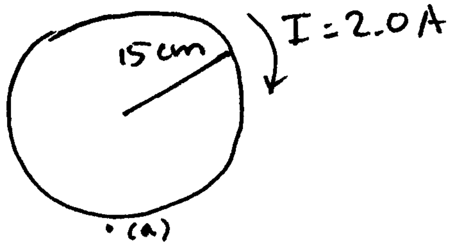
$$\Rightarrow \quad B \approx \frac{\mu_0 I a^2}{2x^3} = \frac{(1.26\times10^{-6}\ N/A^2)(2.0\ A)(0.15\ m^2)}{2(0.60\ m)^3} = 1.3\times10^{-7}\ T$$

$= 0.13\ \mu T$

Direction: into page

Dipole field: gets weak fast as go away!

A circular wire loop of radius 15 cm and negligible thickness carries a 2.0 A current clockwise as viewed from above. Use suitable approximations to find the magnetic field of this loop (a) in the plane of the loop, 1.0 mm outside the loop, (b) on the loop axis, 60 cm above the center of the loop.

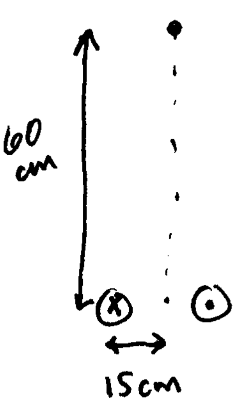


(a) $B = \frac{\mu_0 I}{2\pi r}$ field of long straight wire: so close to the loop compared to its radius, loop looks like a long straight wire
(nearest segment of loop contributes most)

$$B = \frac{(1.26\times10^{-6}\ N/A^2)(2.0A)}{2\pi\,(1.0\times10^{-3}\ m)} = 4.0\times10^{-4}\,T = 400\times10^{-6}\,T = 400\ \mu T$$

$\mu T = 10^{-6} T$

(b) side view:



What result should we use?

Can we use $B \approx \frac{\mu_0 \mu}{2\pi x^3}$? compare to $B = \frac{\mu_0 \mu}{2\pi(x^2+a^2)^{3/2}}$

we have $\left.\begin{array}{l} x = 60\text{ cm} \\ a = 15\text{ cm}\end{array}\right\}$ $x = 4a$ so $x^2 = 16a^2$

Leaving out the a^2 in (x^2+a^2) gives an error of $\sim 1/16 = 6.25\%$ (and then is raised to 3/2)

Whether this is OK depends on your purpose!

Use approx:

$\mu = I\pi a^2 = (2.0A)\pi(0.15m)^2 = 0.14\ A\cdot m^2$

$$\Rightarrow B \cong \frac{(1.26\times10^{-6}\ N/A^2)(0.14\ A\cdot m^2)}{2\pi\,(0.60\ m)^3} = 0.13\times10^{-6}\,T$$

~~[illegible]~~

If you do the exact calculation you get $0.12\times10^{-6}\,T$

(off by ~~[illegible]~~ 10% due to raising denominator to 3/2 power)

12:25 Showed mag dipoles produce $\vec{B}$ analogous to electric dipole $\vec{E}$

What about force & torque?

Force: Remember electric dipoles: experience no force in uniform $\vec{E}$ b/c forces on ⊕ & ⊖ then cancel

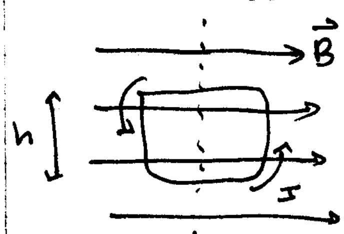
Likewise we found last time that there is no magnetic force on a loop of current in uniform $\vec{B}$

BUT in a nonuniform field, a dipole feels a force

Torque Let's see how magnetic dipoles respond to $\vec{B}$ in terms of rotation

$\boxed{\text{CT}}$ force on top = force on bottom side = 0

b/c top, bottom sides $\parallel$ to $\vec{B}$



Using $\vec{F}_{\text{by } \vec{B} \text{ on } I} = I\vec{L}\times\vec{B}$

$\vec{F}_{\text{on left}} = IhB$ out of page

$\vec{F}_{\text{on right}} = IhB$ into page

$\Rightarrow$ rotates around dotted axis w/ left side coming forward, right side going back

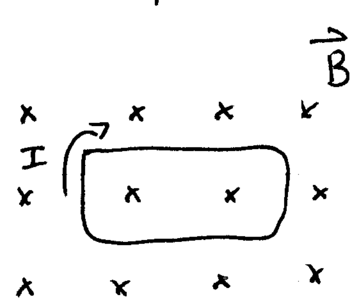
$\Rightarrow$ torque rotates loop

Direction of $\vec{\mu}$ is same as $\vec{B}$ of loop (not the $\vec{B}$ drawn which is exerting the forces / torque): initially out of page

$\Rightarrow$ Loop rotates to bring $\vec{\mu}$ parallel to the external $\vec{B}$ (the one causing the torque): once loop has rotated, $\vec{\mu}$ points to the right

Show using 3D props

$\boxed{\text{CT}}$

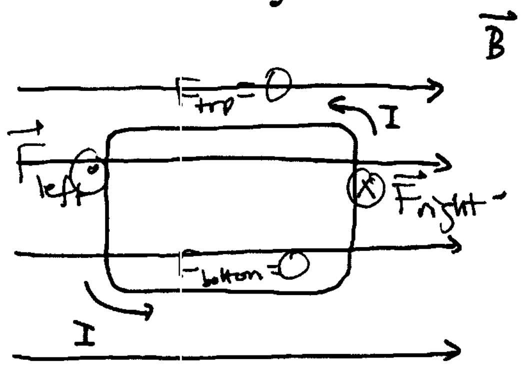


In this case loop does not rotate — forces all act in plane of loop $\rightarrow$ no torque

$\vec{\mu}$ is already parallel to $\vec{B}_{ext}$

A current loop is placed in a magnetic field as shown below. Which way does the loop rotate?

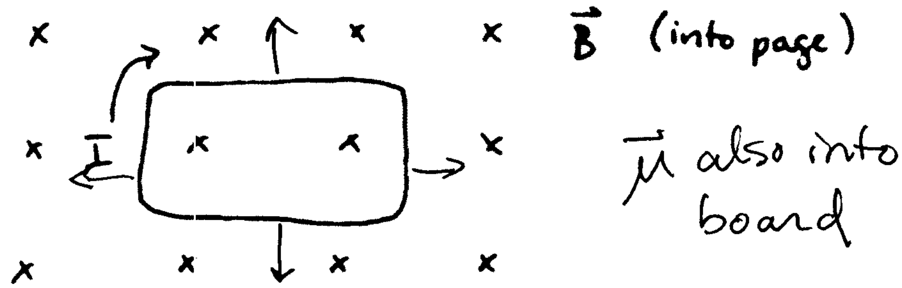
$I\vec{L}\times\vec{B}$:



end up with $\vec{\mu}$ turning to be $\parallel$ to $\vec{B}$

1. The left side comes forward and the right side goes backward.
2. The left side goes backward and the right side goes forward.
3. The loop rotates in the direction of the current.
4. The loop rotates opposite to the direction of the current.
5. The loop does not rotate — there is no net torque.

A current loop is placed in a magnetic field as shown below. Which way does the loop rotate?



1. Left side forward/right side backward.
2. Left side backward/right side forward.
3. The loop rotates in the direction of the current.
4. The loop rotates opposite to the direction of the current.
5. The loop does not rotate — there is no net torque.

With $\vec{\mu}$ defined as before, can show

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

which is just like the electric case: $\vec{\tau} = \vec{p} \times \vec{E}$

We didn't worry much about torques on electric dipoles — but torques on magnetic dipoles are very important:

- magnetic sensing
- NMR, MRI
- electric motors