Announcements 2/25/10

In addition to Thursday evenings, I can schedule a Friday afternoon or Monday afternoon time to go over general questions about the previous two classes. Email me if interested.

Lab this week: Please wear a short-sleeved shirt (we will be doing electrocardiography). (Layering is fine!) Lab materials are all available in lab.

PS6 will be posted this afternoon.

Reading:

Today: Wolfson 24.2, 24.3 ("Plasmas", "semiconductors," and "superconductors" are optional —you may find them interesting!), and 24.4.

Optional additional reading on batteries posted on web site.

[24.5 is assigned as preparatory reading for lab after break.]

Thursday: 25.1-25.2. I encourage you to read before class.

Key ideas from last time

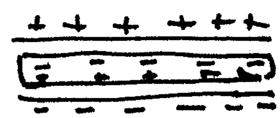
Dielectrics:

- When source charges of the field are fixed, electric field is reduced due to polarization of the dielectric
- When potential difference across a capacitor or cell membrane is fixed, more charge is placed on capacitor plates/ionic layers around cell membrane to maintain same electric field

In both cases:

- capacitance increases: $C = \kappa C_0$ (C_0 is without dielectric) $C = \frac{Q}{\Delta V_{cap}}$
- electric field is less than the electric field of the source charges alone:

$$E_{total} = E_{sources}/\kappa$$



$E_{total} = \frac{E_{plates}}{\kappa}$

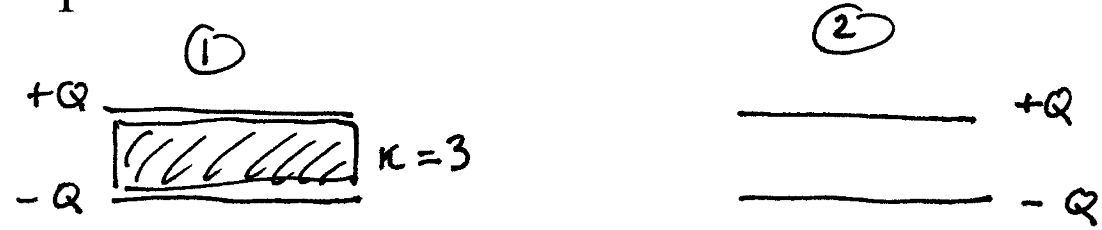
b/c polarized dielectric has an opposing field

Potential energy of point charges in water (or other dielectric) is reduced by factor of κ

Energy in capacitor is still $U_{cap} = Q\Delta V_{cap}$, so whether energy increases or decreases depends on situation

Will finish discussing electrical signals from brain and heart in lab

Two identical capacitors are each given charge ±Q on their plates and then disconnected from their batteries. A dielectric with κ = 3 is inserted between the plates of capacitor 1. Capacitor 2 is filled with air. Compare the energies stored in capacitors 1 and 2:



1. $U_1 = U_2$
2. $U_1 = 3U_2$
3. $U_1 = 9U_2$
4. $U_1 = U_2/3$
5. $U_1 = U_2/9$
6. Need more information

$U_{cap} = \frac{1}{2} Q \Delta V_{cap}$

$\Delta V_{cap} = E_{total}\, d$

so ΔV_{cap} goes down by κ

1 $U_{cap} = \frac{1}{2} Q \frac{\Delta V_{cap,2}}{3} = \frac{U_2}{3}$

$U_{cap} = \frac{1}{2} C \Delta V_{cap}^2$

C has gone up by κ $C_1 = \kappa C_2$

ΔV_{cap} has gone down by κ $\Delta V_{cap,1} = \frac{\Delta V_{cap,2}}{\kappa}$

$U_{cap,1} = \frac{1}{2}(\kappa C_2)\left(\frac{\Delta V_{cap,2}}{\kappa}\right)^2 = \frac{U_{cap2}}{\kappa}$

$U_{cap} = \frac{1}{2}\frac{Q^2}{C}$ $C\uparrow \rightarrow U\downarrow$

3/2/2010

Batteries: how maintain potential difference between terminals?

two different metals in an ionic solution (water with some kind of ions dissolved in it - usually an acid)

different chemical rxns occur at the two metals

- one uses chemical energy to keep adding ⊖ electrons to ⊖ electrode
- ~~other~~ other uses chemical energy to remove e^- from the ⊕ electrode

Conservation of energy:

reaction ~~releases an amount of~~ reduces chemical energy U_{chem} by some amount equal to the gain in electric potential energy

$$\Delta U_{chem} + \Delta U^E = 0$$ (no other forms of energy changing

change in electric potential energy:

move ⊖ electron from ⊕ to ⊖ terminal

$$q\Delta V = \Delta U^E = (-e)(-\Delta V_{batt}) = e\Delta V_{batt}$$

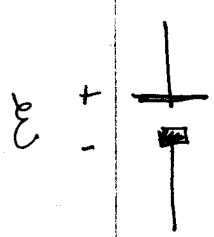
+ same as if imagine moving a proton from ⊖ to ⊕

Demo: lemon battery

Batteries run down when deplete chemicals needed to keep rxn going

Source of emf $\mathcal{E}$

We call anything that converts non-electrical energy to electrical potential energy a source of emf



symbol for a source of emf

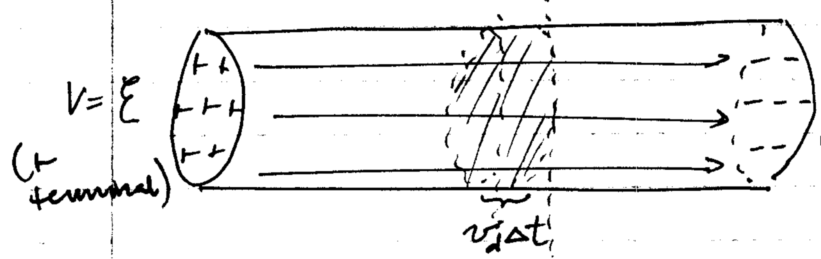
keeps terminals at fixed potential difference $\mathcal{E}$

Current flow in conductors

(1) How much current for a given $\mathcal{E}$?
(2) How do we get back the energy given to the mobile charges by the battery?

Electrostatic equil: conductor is an equipotential
With a battery and a conducting path for charge to follow, not at equilibrium!

Attach battery terminals to ends of tube filled with salt solution



ends like charged sheets $\Rightarrow$ uniform $\vec{E}$ inside

⊕ ions move toward ⊖ end, ⊖ ions toward ⊕ end

Turns out that in a conductor the average speed of the moving charges is constant — "friction" causes moving charges to steadily lose energy in form of heat

call average speed v_d; ⊕ if toward $V=0$, ⊖ in opp direction
avg speed det by $\vec{E}$

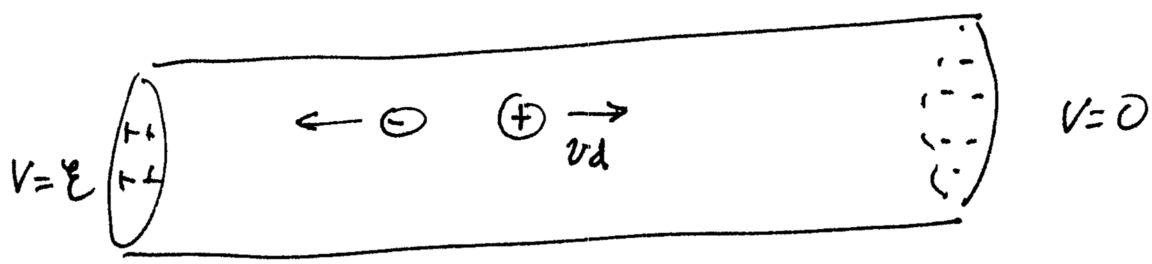
Then current $I = \dfrac{\Delta Q}{\Delta t} = \dfrac{\text{amt of charge passing dashed line in } \Delta t}{\Delta t}$

- volume of chunk $= A v_d \Delta t$
  charge in it $= nqAv_d \Delta t$

$\Rightarrow I = nqAv_d$

CT Net current from ⊕ & ⊖ ions is double
⊖ ions move opp way!

A voltage is applied across a tube of salt solution containing equal densities of moving charges with charges +e and –e. The absolute value of the drift speeds is the same for both types of moving charge. The total current is



1. Zero
2. Twice as much as is due to just the positive charges.
3. Less than the current due to just the positive charges, but not zero.

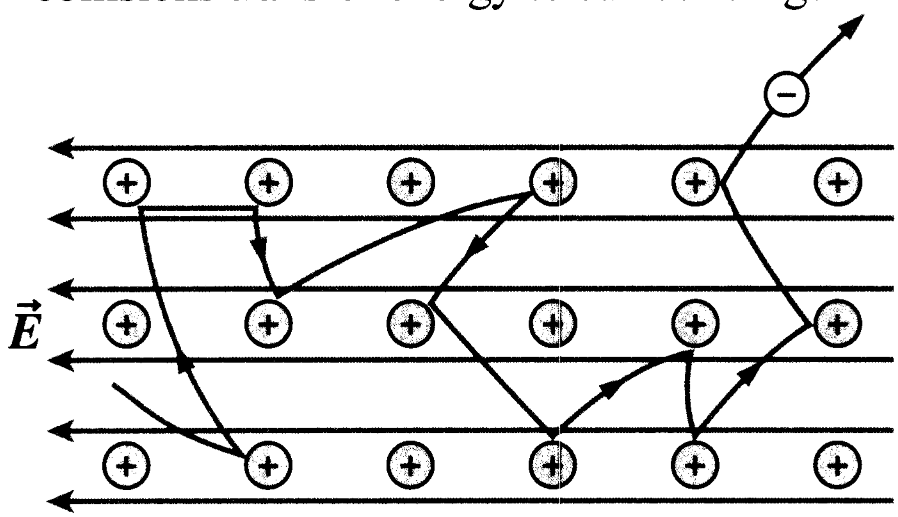
$$I_{total} = I_+ + I_-$$
$$= neAv_d + n(-e)A(-v_d)$$
$$= 2neAv_d$$

Think of v_d as being positive if in the same direction as $\vec{E}$, negative if opposite $\vec{E}$

v_d

What determines average speed? Why is it constant?

- Instead of accelerating steadily, mobile charges in a conductor have constant average speed due to frequent collisions with surroundings
- metal: moving electrons collide with positively charged nuclei
- ionic solution: moving ions (positive or negative) collide with solvent molecules or other ions
- collisions transfer energy to surroundings — heats it up



$v_d = \overline{v} = a\tau$

average speed

average time between collisions

$a = \frac{qE}{m} \Rightarrow v_d = \frac{qE\tau}{m}$

so take-home point: $v_d = E$

Suppose you want the proteins to move faster in your gel so that your experiment will be done sooner. You decide to double the voltage across the gel. However, you don't want to double the current in the ionic solution to minimize heating up the gel. How could you keep the current the same while doubling the voltage?

1. Dilute the solution to half the original concentration.
2. Make the gel twice as long from top to bottom.
3. Reduce the cross-sectional area of the gel to half its original area.
4. Both 1 and 2.
5. Both 1 and 3.
6. Both 2 and 3.
7. All of 1, 2, and 3.

decrease by half

doubled

$$I = nqAv_d$$

<u>What determines v_d?</u>

mobile charges move in random directions b/c of thermal motion
repeatedly collide w/surroundings and change direction
$\vec{E}$ gives a ⊕ charge ~~an~~ acceleration parallel to $\vec{E}$
⊖ charge an acceleration opposite $\vec{E}$

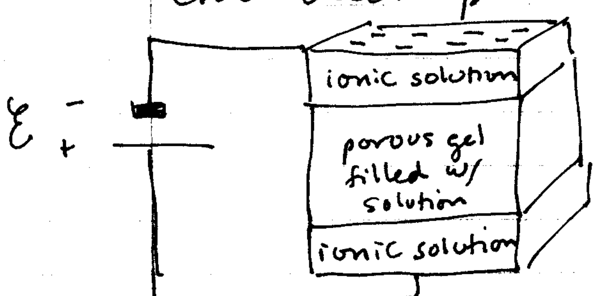
but charges only go in that direction briefly before the next collision redirects them

v_d = average speed $\bar{v} = a\tau$ ← τ = average time between collisions
(↑ average)

$a = \frac{qE}{m} \Rightarrow v_d = \frac{qE}{m}\tau$

Increasing E increases v_d

Gel electrophoresis:



Apply potential difference →
top becomes ⊖ terminal
(like ⊖ charged plate)

bottom becomes ⊕ terminal
(like ⊕ charged plate)

Treat proteins → highly ⊖ charged
SDS-PAGE: $\frac{q}{m}$ is same for all due to surfactant molecules decorating
also all are denatured → long rodlike shape
just vary by length
so qualitatively τ increases with decreasing size
τ decreases with increasing size
⇒ drift speed v_d is greater for smaller size
(may not be linear?)

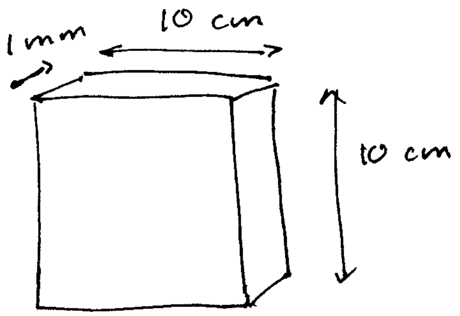
Current in gel is primarily from ionic solution

$\boxed{CT}$ Increase V, want to keep I the same — what do we change?

Reducing A, n both reduce $I = n q v_d A$
If ~~reduce~~ double L that will also reduce I —
but reduces E as well
$\mathcal{E} = EL$
so doubling $\mathcal{E}$ and L ⇒ same E!

Problem: find drift speed in gel

You are running current through a gel that is 10 cm tall, 10 cm wide, and 1 mm thick, filled with 0.01 mol/L NaCl solution (6.023×10^{24} ions/m^3). The voltage from the top to the bottom of the gel is 110 V and the current in it is 100 mA. Find the drift speed of the ions in the gel.



$$I = I_+ + I_-$$

Showed before that current from ⊕ and ⊖ is same (if ions have same drift speed — roughly reasonable here)

$$m_{Na^+} \approx m_{Cl^-}$$

23 amu vs 35 amu

If approximate drift speeds as same:

$$I = 2I_+ = 2nqAv_d$$

$$\Rightarrow \quad v_d = \frac{I}{2nqA} = \frac{100\times10^{-3}\,A}{2(6.023\times10^{24}\,\text{ions}/m^3)(1.6\times10^{-19}\,C)(0.10\,m)(0.001\,m)}$$

$$v_d = 5.2\times10^{-4}\ m/s$$

If want to allow for different masses: (more elaborate than intended)

$$\left.\begin{aligned} I_+ &= n\,e\,A\,v_{d,Na^+} \\ I_- &= n(-e)A\,v_{d,Cl^-} \end{aligned}\right\}$$

and because $v_d = \frac{q}{m}E\tau$ if we assume τ still same then can consider

$$v_{d,Na^+}\,m_{Na^+} = -v_{d,Cl^-}\,m_{Cl^-}$$

$$v_{d,Na^+}(23) = -v_{d,Cl^-}(35)$$

$$\Rightarrow \quad v_{d,Cl^-} = -\frac{23}{35}v_{d,Na^+}$$

so $I = neAv_{d,Na^+} + n(-e)A\left(-\frac{23}{35}\right)v_{d,Na^+}$

and then substituting values gives

$$v_{d,Na^+} = 6.3\times10^{-4}\ m/s$$

How much current for a given ΔV ?

"Ohmic materials": observe experimentally

$$\vec{J} = \frac{I}{A} \text{ in direction of } \vec{E} = \sigma \vec{E} = \frac{\vec{E}}{\rho}$$

σ: conductivity: property of material

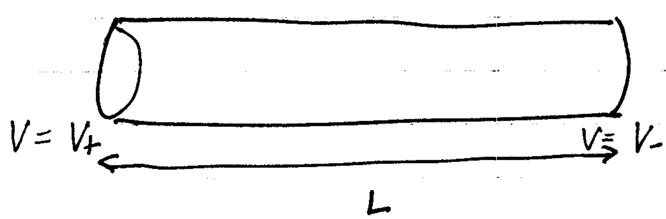
ρ: resistivity $\rho = \frac{1}{\sigma}$

$\boxed{\text{can show } \sigma \propto v_d}$

$\rho \propto \frac{1}{v_d}$

$$\Rightarrow I = \frac{AE}{\rho}$$

In a conductor of constant cross-sectional area, $\vec{E}$ is constant & parallel to direction of current flow



$|\Delta V_{cond}| = V_+ - V_-$ magnitude of pot diff (often just notated V)

$|\Delta V_{cond}| = EL$

CT Two pieces of conductor of same ΔV_{cond}, A, different L – how does I compare?

I greater in shorter piece:

$$I = \frac{A}{\rho} \frac{|\Delta V_{cond}|}{L}$$

Usually write this relnship in terms of ΔV_{cond}:

$$|\Delta V_{cond}| = I \rho \frac{L}{A} = I R \quad \text{with } R = \text{resistance} = \rho \frac{L}{A}$$

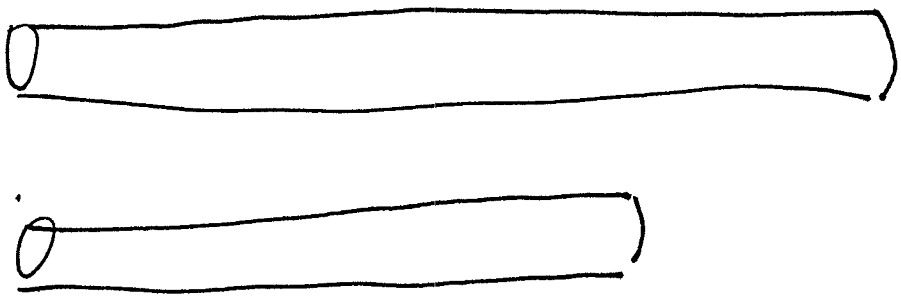
$|\Delta V_{cond}|$ often written as V: remember means pot diff across conductor; also often just used to relate magnitudes

We'll get to how to handle signs next time

Says amount of current increases if R decreases

- bigger A → more current (more ions can get through)
- shorter L → more current
- no/zero / lower ρ → more current

Consider applying a 10 V potential difference across each of two pieces of the same material (therefore same ~~conduc~~resistivity ρ) with the same cross-sectional area and different lengths, as shown.



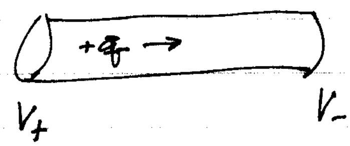
In which piece is the current greater?

1. The longer piece.
2. The shorter piece.
3. The current is the same in both.

How much energy goes into heat?

As charge crosses conductor, loses electric PE

$$\Delta U^E = q(V_- - V_+)$$
$$= -q\,\Delta V_{cond}$$



If many charges move, lose total energy $\qquad V_+ - V_- = \Delta V_{cond}$

$$\Delta U^E = -Q_{total}\,\Delta V_{cond}$$

Rate of converting energy to heat:

$$\text{power} = \frac{\text{heat energy}}{\text{time}} = \frac{\Delta Q_{total}\,\Delta V_{cond}}{\Delta t}$$

where ΔQ_{total} = amt of charge in time Δt

$$P = I\,\Delta V_{cond}$$

increasing ΔV requires reducing I to keep P const

In terms of resistance: can eliminate I or ΔV

$$P = \left(\frac{\Delta V_{cond}}{R}\right)\Delta V_{cond} = \frac{\Delta V_{cond}^2}{R}$$

$$\boxed{\text{increasing resistance} \rightarrow \text{less power for fixed } \Delta V_{cond}}$$

OR $\quad P = I(IR) = I^2R$