Announcements 2/4/10

For next week's self-test, can look at book if you get stuck

Tonight's problem session: I will be present, can go over lingering optics questions or answer questions about electrostatics in general as well as problem set.

Reading for next week:
Tuesday sections 20.5 and 21.1
Thursday section 21.6 p. 359- end of second line on p. 360

We are not going to cover Gauss's Law (most of Ch. 21). So, you are not responsible for anything concerned with Gauss's Law.

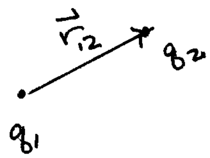
[PS 3 Add'l problem 2
Due Thurs with self-test]

Key ideas from last time

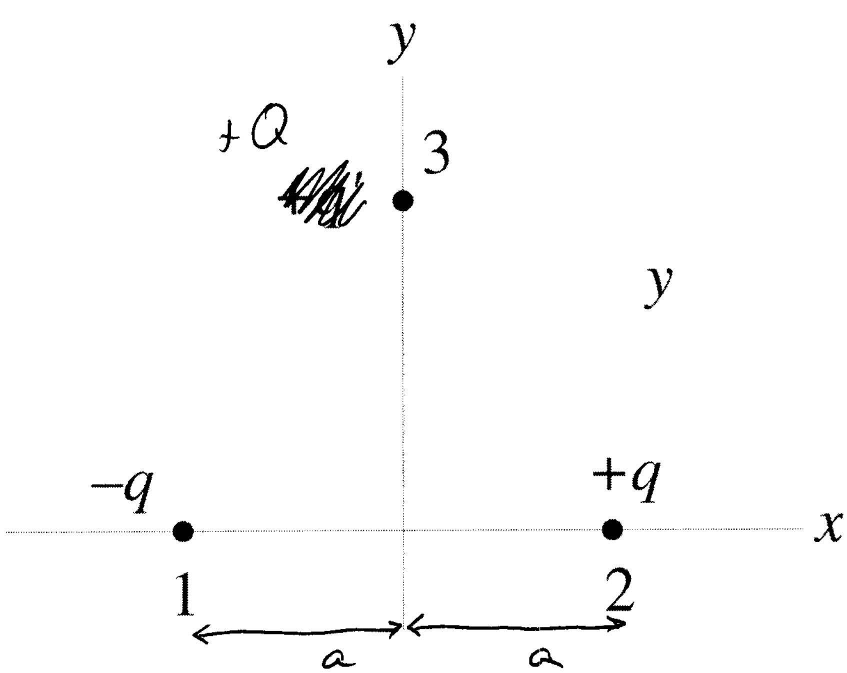
Electrical forces between charged objects:
- increase as the total amount of charge increases
- decrease as the distance between objects increases
- attractive for oppositely charged objects, repulsive for like

Pointlike charged particles: force acts along the line between the particles

$$\vec{F}_{by\,1\,on\,2} = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12}$$



In the arrangement of charge shown, charges 1 and 2 have equal magnitudes and opposite sign, and they are placed equal distances from the origin.
What is the direction of the vector sum of the forces exerted by 1 and 2 on 3?



1. The +x direction
2. The –x direction
3. The +y direction
4. The –y direction
5. Another direction
6. The vector sum of the forces on 3 is zero.

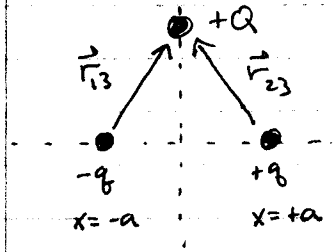
J.J.

2/4/2010

Last time considered <u>dipolar</u> arrangement of charge – very important

b/c for reasons we will discuss next week, dipole is model for <u>many</u> important systems!

Calculating $\vec{F}_{on\,3}$ from last time:



$\vec{r}_{13} = a\hat{i} + y\hat{j}$

magnitude is $r_{13} = \sqrt{a^2+y^2}$

Unit vector $\hat{r}_{13}$:

divide vector $\vec{r}_{13}$ by magnitude

$$\Rightarrow \hat{r}_{13} \equiv \frac{a\hat{i}+y\hat{j}}{\sqrt{a^2+y^2}} = \frac{a}{\sqrt{a^2+y^2}}\hat{i} + \frac{y}{\sqrt{a^2+y^2}}\hat{j}$$

To find $\vec{r}_{23}$: Same procedure, now horizontal component is in opposite direction

$$\Rightarrow \vec{r}_{23} = -a\hat{i} + y\hat{j} \quad \text{and} \quad \hat{r}_{23} = -\frac{a}{\sqrt{a^2+y^2}}\hat{i} + \frac{y}{\sqrt{a^2+y^2}}\hat{j}$$

Add forces together:

$$\vec{F}_{13} = \frac{k(-q)Q}{r_{13}^2}\hat{r}_{13} = -\frac{kqQ}{(a^2+y^2)}\frac{a\hat{i}+y\hat{j}}{\sqrt{a^2+y^2}} = -\frac{kqQ}{(a^2+y^2)^{3/2}}(a\hat{i}+y\hat{j})$$

and likewise

$$\vec{F}_{23} = \frac{kqQ}{r_{23}^2}\hat{r}_{23} = \frac{kqQ}{(a^2+y^2)^{3/2}}(-a\hat{i}+y\hat{j})$$

Adding them: y-components cancel, x-components add

$$\Rightarrow \vec{F}_{on\,3} = \vec{F}_{13} + \vec{F}_{23} = -\frac{2kqQa}{(a^2+y^2)^{3/2}}\hat{i}$$

notice force weaker than just force of a point chg

11:35 $\boxed{\text{CT}}$ Ranking task (or square of charges? was on website)

All electrical forces can be calculated this way!

But can imagine it gets tedious if there are a lot of charges involved.

Also, there is a more fundamental problem — <u>how</u> do these charges exert force on each other?

Gravitational field

How can Earth exert a force on massive objects without touching them?

Explanation: Earth creates a "gravitational field" that fills the space around it

- that field exists everywhere, regardless of whether there is an object to feel it
- it has both a strength and a direction that ~~let you~~ determine the strength and direction of forces it exerts
- it must be a property of the source of the field only — i.e. Earth!

Field describes effect of ~~Earth~~ source only

"Moon's gravity is 1/6 that of Earth" - tells you the effect of the Moon or the Earth without ~~knowing~~ needing to specify mass of object in its vicinity

Defining grav field

Choose y axis to be vertical → Near Earth's surface $\vec{F}_g = -mg\hat{j}$

Separate force into effect of source and property of object experiencing force:

~~Mass~~ $$\vec{F}_g = \underbrace{(-g\hat{j})}_{\text{source}}\underbrace{m}_{\text{object feeling force}} = \vec{g}m \quad \text{with } \underbrace{\vec{g} \equiv -g\hat{j}}_{\text{grav. field}}$$

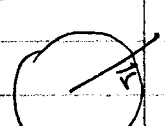
$\boxed{\text{force per mass same for any object}}$

Tell them to draw it

$\vec{g}$ is a vector at every point in space

doesn't depend on position b/c of choosing approx near Earth

Use full equation for grav:



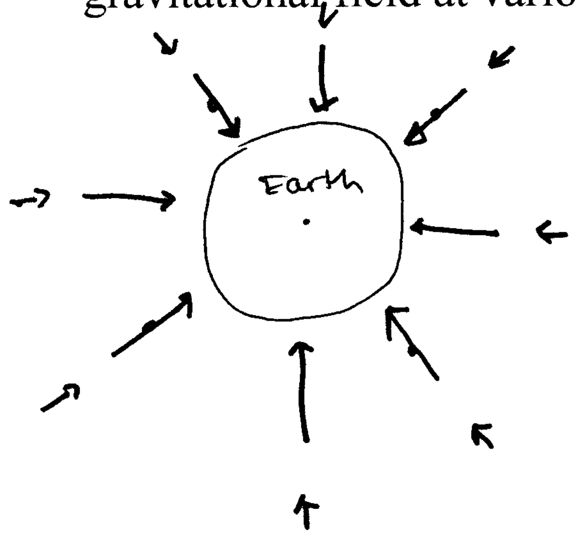
$$\vec{F}_g = -\frac{mMG}{r^2}\hat{r} = m\left(-\frac{M_E G\hat{r}}{r^2}\right) = m\vec{g}(\vec{r}) \text{ with } \vec{g} \equiv -\frac{M_E G\hat{r}}{r^2}$$

origin @ ctr of Earth

$\vec{r}$ from ctr to point where m is

Now $\vec{g}(\vec{r})$ depends on location!

Sketch vectors indicating the magnitude and direction of the gravitational field at various points around the Earth.



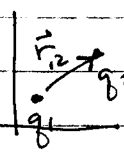
field reprented by vector at every point in space

different directions, magnitudes dep. on location

Electric field

Same idea: charged objects fill space around themselves with an electric field
- this field exerts forces on other charges

Separate electric force into effect of source and charge of object feeling force:



$$\vec{F}_{\text{by 1 on 2}} = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12} = \underset{\text{object feeling force}}{q_2}\underbrace{\left(\frac{kq_1}{r_{12}^2}\hat{r}_{12}\right)}_{\text{effect of source at location of object feeling force}} = q_2\vec{E}_1(x_2,y_2,z_2)$$

Now clearly $\vec{E}_1 \equiv$ electric field of source (#1) depends on location:

$$\vec{E}_1(x_2,y_2,z_2) \equiv \frac{kq_1}{r_{12}^2}\hat{r}_{12}$$

$\boxed{\text{Force per charge that an object placed at } (x_2,y_2,z_2) \text{ feels from source (\#1)}}$

Show Physlet: what $\vec{E}$ looks like
- points away from ⊕ charge, toward ⊖ charge
- same direction & ~~strength~~ distance dep as force on ⊕ chg
  opp direction as force on ⊖ chg

$\vec{F}_{12} = q_2\vec{E}_1(x_2,y_2,z_2)$ means if q_2 ⊖, $\vec{F}_{12}$ is in opp direction

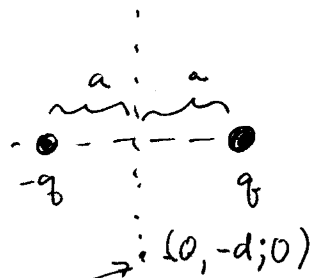
Handout shows 4 benchmark arrangements of charge
Nearly everything can be considered w/these

Fields from multiple source charges

Because forces add like vectors, so do fields:

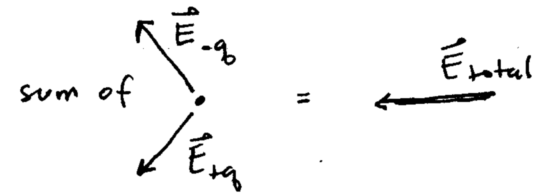
$$\vec{E}_{total}(x,y,z) = \vec{E}_1(x,y,z) + \vec{E}_2(x,y,z)$$ is combined field of charges 1 & 2 at location (x,y,z)

12:12 $\boxed{CT}$ Direction of dipole $\vec{E}$ at $(0,-d,0)$?



a point (call it P) : NO CHARGE HERE

Vector sum of $\vec{E}_{-q}(0,-d,0)$ and $\vec{E}_q(0,d,0)$
- same magnitudes
- directions as drawn



Calculate $\vec{E}$ by same process as force earlier in class
General result for any point on y-axis is

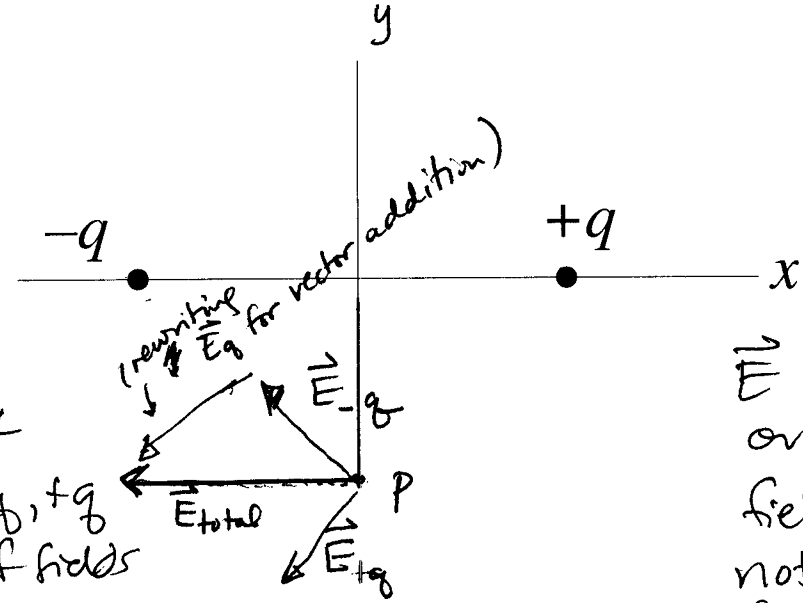
$$\vec{E}_{dipole}(0,y,0) = -\frac{2kqa}{(a^2+y^2)^{3/2}}\hat{i}$$ if dipole is made of charges $\pm q$ at $x = \pm a$

Limit as you go far away: $y \gg a \Rightarrow \vec{E}_{dipole}(0,y,0) \approx -\frac{2kqa}{y^3}\hat{i}$

Field strength $\propto \frac{1}{y^3}$ — decreases faster w/distance than just a single charge (just like force)

$\boxed{CT}$ Direction of dipole $\vec{E}$ on dipole axis?
(Ask about both sides)

An electric dipole is placed as illustrated. What is the direction of its electric field at point P?



P is same dist away from −q, +q so strengths of fields are same

$\vec{E}$ depends only on sources of field (dipole) not on anything feeling field.

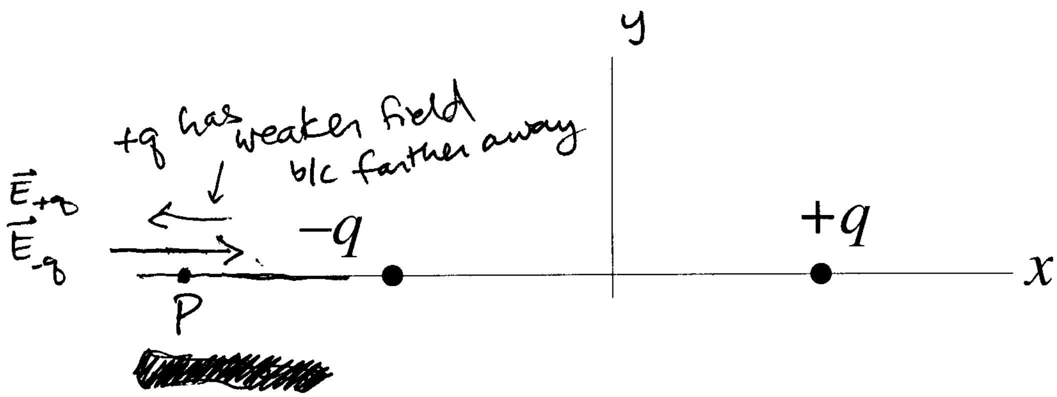
1. The +x direction
2. The –x direction CORRECT
3. The +y direction
4. The –y direction
5. Either +x or -x, depending on whether the charge at point P is positive or negative

True of force, not true of $\vec{E}$

6. Either +y or -y, depending on whether the charge at point P is positive or negative
7. The electric field at P is zero.

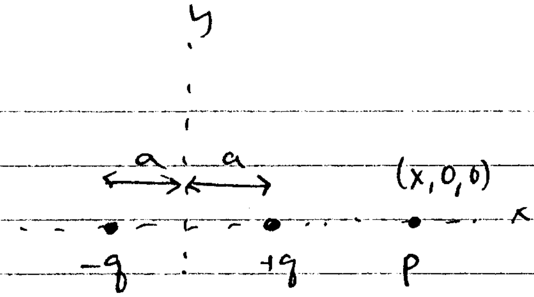
∴ sum is horizontal

An electric dipole is placed as illustrated. What is the direction of its electric field at point *P*?



1. The +x direction
2. The –x direction
3. The +y direction
4. The –y direction
5. Either +x or -x, depending on whether the charge at point *P* is positive or negative
6. Either +y or -y, depending on whether the charge at point *P* is positive or negative
7. The electric field at *P* is zero.

Calculate $\vec{E}(x,0,0)$ on x-axis
(Choose positive x for simplicity)



$$\vec{E}_{dipole}(x,0,0) = \vec{E}_+(x,0,0) + \vec{E}_-(x,0,0)$$

$$= \frac{kq}{r_{+P}^2}\hat{r}_{+P} + \frac{k(-q)}{r_{-P}^2}\hat{r}_{-P}$$

Distance from $+q$ to P is $x-a$
Distance from $-q$ to P is $x+a$
Directions of $\hat{r}_{+P}$ and $\hat{r}_{-P}$ are both $+\hat{i}$
(unit vector pointing from $+q$ to P is in $+\hat{i}$ direction, likewise unit vector from $-q$ to P is also in $+\hat{i}$)

Substituting:

$$\vec{E}_{dipole}(x,0,0) = \frac{kq}{(x-a)^2}\hat{i} + \frac{k(-q)}{(x+a)^2}\hat{i}$$

$$= kq\hat{i}\left\{\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right\}$$

Algebra gives

$$\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2} = \frac{4ax}{x^4 - 2a^2x^2 + a^4}$$

so

$$\vec{E}_{dipole}(x,0,0) = kq\hat{i}\left\{\frac{4ax}{x^4 - 2a^2x^2 + a^4}\right\}$$

and if $x \gg a$ we can drop all terms except x^4 in the denominator

$$\Rightarrow \vec{E}_{dipole}(x,0,0) \cong kq\hat{i}\left\{\frac{4ax}{x^4}\right\} = \frac{4akq}{x^3}\hat{i}$$

for $x \gg a$

So far away from the dipole, strength of electric field $\propto \frac{1}{\text{distance}^3}$ in any direction while for a single point charge,

$$\text{strength of field} \propto \frac{1}{\text{distance}^2}$$

* Field strength decreases more rapidly for dipole than for single charge *

Next benchmark shape: spherically symmetric
go to page of notes

Can prove via fairly involved integration (I'll post a handout for those interested) or by method called Gauss's Law. We'll use this result w/o proof.

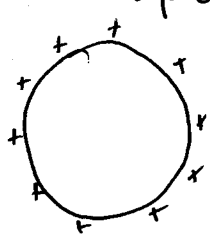
Plausible — if spherically symmetric, field has to be radial so that field pattern is also sph. symm.
So only possible diff would be w/amt of chg.

up to here

Van de Graaf generator:

Inside the generator rubber belts rub on some other material – ~~charge is placed on~~ get charged

Belt moves up to metal dome
When belt comes in contact w/dome, charge is free to move on metals (conductors) so it spreads uniformly over surface



$$\Rightarrow \vec{E}(r) = \frac{kQ}{r^2}\hat{r} \quad \text{for } r > \text{radius of dome}$$
$Q \equiv$ total charge on dome

$\boxed{CT}$ about field strengths @ different distances

$\boxed{\text{Problem}}$: amount of charge on dome

$$E(r = 10\ \text{cm}) = \cancel{\frac{kq_{dome}}{(0.30\ m)^2} = 3\times10^6\ N/C} \quad \frac{kq_{dome}}{r^2} = 3\times10^6\ N/C$$

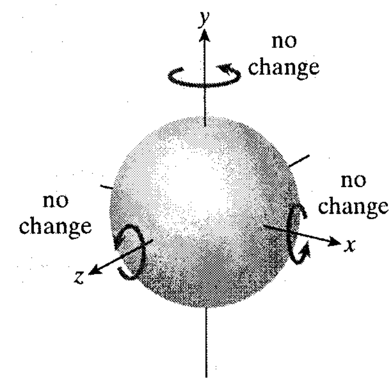
$$\left.\begin{array}{l} k = 9.0\times10^9\ Nm^2/C^2 \\ r = 0.10\ m \end{array}\right\} \Rightarrow q_{dome} = \frac{(3\times10^6 N/C)(0.10 m)^2}{(9\times10^9\ Nm^2/C^2)}$$

$$q_{dome} = 3.3\times10^{-6}\ C = 3.3\ \mu C$$

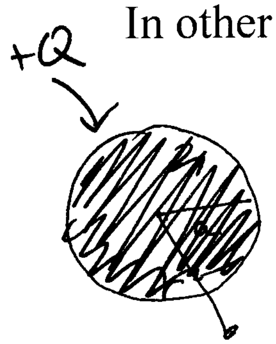
A lot: e^- charge is $1.6\times10^{-19}\ C$!

The electric field of any spherically symmetric distribution of charge, *outside* that charge distribution, is the same as the field of a pointlike charged particle with the same charge located at the center of that distribution.

Spherical Symmetry



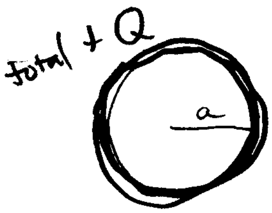
In other words:



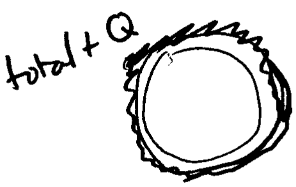
A uniformly charged solid sphere of radius a and total charge Q has electric field

$$\underline{\vec{E}_{sphere}(r) = \frac{kQ}{r^2}\hat{r}} \qquad \text{for } r > a$$

where r is measured from the **center** of the sphere.



A solid sphere of radius a and total charge Q with all the charge distributed uniformly over the outer surface has **the same field** for $r > a$ (outside the sphere)



A hollow uniformly charged spherical shell of outer radius a and total charge Q has **the same field** for $r > a$ (outside the shell).

As long as it has spherical symmetry, the field is the same outside!