Announcements 2/25/10

Tonight's problem session: I might not make it depending on the weather, but Elizabeth will be there (?).

In addition to Thursday evenings, I can schedule a Friday afternoon or Monday afternoon time to go over general questions about the previous two classes. Email me if interested.

Lab next week: Please wear a short-sleeved shirt (we will be doing electrocardiography). (Layering is fine!)

Reading:

Feb 23: 23.2, 23.3 up to "Got It?" 23.1 (skipping p. 389 and example 23.3; we are doing much more with dielectrics)

Today: 23.4 but *not* example 23.5; 24.1

March 2: Wolfson 24.2, 24.3 ("Plasmas", "semiconductors," and "superconductors" are optional —you may find them interesting!), and 24.4. I encourage you to read at least some of this before class.

Comments on pacing questions

J.J. 2/25/2010

1. Dielectrics vs conductors
2. Effect of dielectric w/ fixed source charges
   - $\vec{E}$, ΔV_{cap}
   - How much $q_{induced}$?
   - C
3. Effect of dielectric w/ fixed potential difference
   - $\vec{E}$, Q_+ and $q_{induced}$
   - C – charge stored on charged cell membrane
4. Energy w/ dielectrics
   - U_{cap}
   - $U_{charges}$ (NaCl problem)
5. Electrical signals ~~cells~~ from charged cell membranes: electrocardiography, electroencephalography

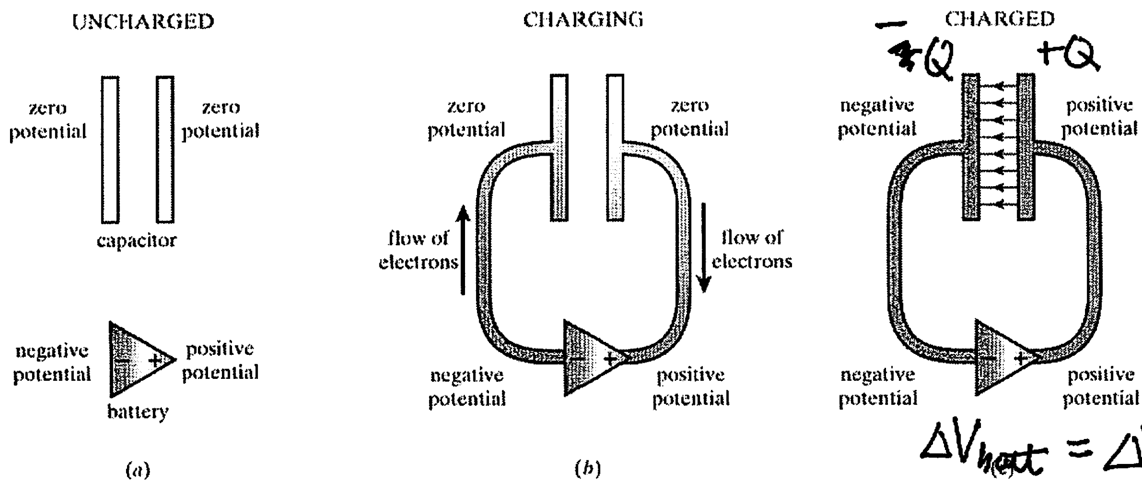
Dielectrics ("polarizable materials") vs conductors

Conductors (metals, ionic solutions): Some e^- completely free to move within metal
$\Rightarrow$ $E = 0$ within metal

Key ideas from last time

Charging a capacitor:

Battery provides potential energy to the charges on its terminals, and thus to the charges on the electrodes



$\Delta V_{batt} = \Delta V_{cap}$

Defined C (capacitance) $Q \equiv C \Delta V_{cap}$

showed for parallel plates $C \equiv \frac{A \epsilon_0}{d}$

A = area of plate

d = dist between

Energy stored $U = \frac{1}{2} Q \Delta V_{cap} = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} C \Delta V_{cap}^2$

C tells how much energy for ΔV_{cap}

Dielectrics vs. conductors

↑ "polarizable material"

Conductor (metal, ionic solution): has e^- free to move anywhere in conductor

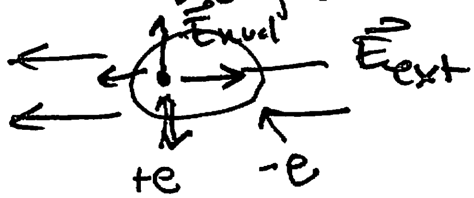
→ $\vec{E}=0$ in e^-static equilibrium (about to consider non equil)

Dielectric: anything else

"insulator": in an $\vec{E}$, charge shifts, but e^- tied pretty tightly to nuclei

$\vec{E}_{ext}$ ~~ ~~ shifts e^- density until force from nuclei balances it

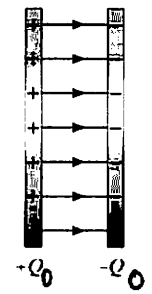
→ e^- feel no net force



Insulator/dielectric always polarizes but only reduces total $\vec{E}$ inside it — does not make $\vec{E}=0$ inside as conductors do b/c e^- can't move far

Electric fields in dielectrics

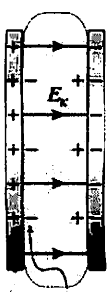
Case 1: Source charges of the original electric field stay the same: E_{total} is reduced
(Examples: Charged capacitor that is not attached to a battery, ions in a dielectric solvent)



No dielectric:
Potential difference across capacitor is $\Delta V_{cap,0}$
Electric field is E_{plates}
Area of plates is A, separation between plates is d
Charge on capacitor plates is $\pm Q_0$, charge density is $\pm\sigma_0 = \pm Q_0/A$

$$E_{plates} = E_0 = \frac{\sigma_0}{\varepsilon_0} = \frac{Q_0}{A\varepsilon_0} = \frac{\Delta V_{cap,0}}{d}$$

Add dielectric: dielectric polarizes, giving induced charge on dielectric surface due to polarization of dielectric. (This charge is sometimes called "bound charge" because it is part of the dielectric, and can't move onto the capacitor. The charge on the capacitor plates is sometimes called "free charge".)



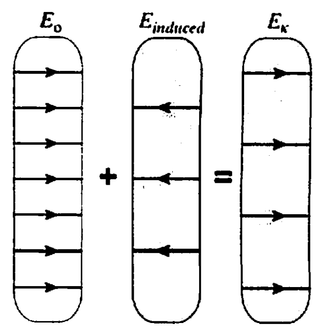
Q_0 $q_{induced}$

Electric field strength (and potential difference) are *reduced* due to the polarization of the dielectric.

The electric field in the dielectric is the combined electric field of the charge on the plates and the induced charge on the dielectric. The charge on the plates hasn't changed, so we have

$$\vec{E}_{total} = \vec{E}_{plates} + \vec{E}_{induced} = \frac{Q_0 + q_{induced}}{A\varepsilon_0} \text{ to the right}$$

(On the figure, the electric field in the dielectric is labeled E_κ. This is the same as E_{total}.)



$q_{induced}$ is the amount of induced charge at the left of the dielectric, adjacent to the positive plate, so that $Q_0 + q_{induced}$ is the total charge in the two layers combined. As the induced charge adjacent to the positive capacitor plate is negative, the total $Q_0 + q_{induced}$ is less than Q_0, corresponding to a reduced electric field:

We define κ, the dielectric constant, as a parameter that describes how much the field is reduced compared to the field that the source charges would produce without the dielectric:

$$E_{total} = \frac{E_0}{\kappa} = \frac{\sigma_0}{\kappa\varepsilon_0} = \frac{Q_0}{\kappa A\varepsilon_0}$$

κ is a property of the particular dielectric material. It is a positive number greater than 1. Note κ is *not* the same as $k = 1/4\pi\varepsilon_0$.

How much charge is induced? Combine the two equations for E_{total}:

$$\frac{(Q_0 + q_{induced})}{A\varepsilon_0} = \frac{Q_0}{\kappa A\varepsilon_0} \qquad \text{which can be solved for } q_{induced} \text{ to give}$$

$$q_{induced} = -Q_0\left(1 - \frac{1}{\kappa}\right) \qquad \text{(We expect } q_{induced} \text{ to be negative, and it is.)}$$

This gives the amount of induced charge on the surface of the dielectric, in terms of the dielectric constant and the amount of source charge (in this case, the charge on the capacitor plates).

This derivation is done for two parallel plates, but the relationship between $q_{induced}$ and Q_0, and between E_{total} and E_0, is the same for any source charge arrangement, as long as the source charges do not change when the dielectric is included.

Case 2: The potential difference and electric field do not change: amount of source charge increases (Examples: Capacitor that is attached to a battery, charged cell membrane that has potential difference maintained by molecular ion pumps)

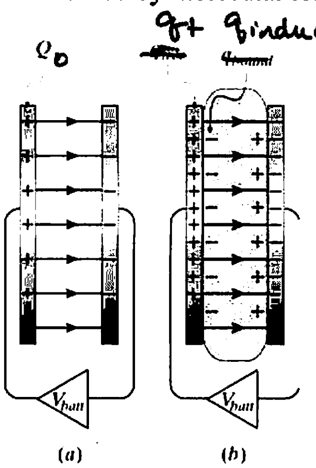


Figure 30.27 Bound and free charges on (*a*) a vacuum-filled and (*b*) a dielectric-filled parallel-plate capacitor connected to a battery.

Now we require ΔV_{cap} and therefore E_{total} to stay the same

Without dielectric: just as in case 1, $E_{plates} = E_0 = \dfrac{\sigma_0}{\varepsilon_0} = \dfrac{Q_0}{A\varepsilon_0} = \dfrac{\Delta V_{cap,0}}{d}$

With the dielectric: The dielectric polarizes, but **the battery puts more charge on the capacitor plates** to maintain the same electric field strength, so now the charge on the positive plate is q_+, and we have

$$\vec{E}_{total} = \vec{E}_{plates} + \vec{E}_{induced} = \vec{E}_0$$

so $$E_{total} = \frac{q_+ + q_{induced}}{A\varepsilon_0} = \frac{Q_0}{A\varepsilon_0}$$

Let us find how much charge is on the plates, q_+, in terms of the original charge Q_0. The last equation gives us

$$q_+ + q_{induced} = Q_0$$

Just as in Case 1, E_{total} is reduced by a factor of κ from the field of just the charged plates:

$$E_{total} = \frac{(q_+ + q_{induced})}{A\varepsilon_0} = \frac{q_+}{\kappa A\varepsilon_0}$$

which we can solve for $q_{induced}$

$$q_{induced} = -q_+\left(1 - \frac{1}{\kappa}\right)$$ (again $q_{induced}$ is negative as we would expect).

Substituting this into $q_+ + q_{induced} = Q_0$ in order to eliminate $q_{induced}$ gives

$$q_+ - q_+\left(1 - \frac{1}{\kappa}\right) = Q_0$$

which can be simplified to

$$\frac{q_+}{\kappa} = Q_0 \qquad \text{or} \qquad q_+ = \kappa Q_0$$

So when ΔV_{cap} is fixed, the amount of charge on each capacitor plate, or on each side of the cell membrane, increases by a factor of κ, compared to what it would be with the same ΔV_{cap} but without the dielectric. **Again this is true for any arrangement of source charges, not just parallel plates.**

In both cases:

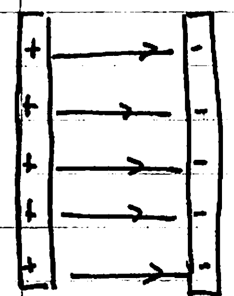
- **The polarized dielectric reduces the electric field by a factor of κ, compared to the field of the source charges alone.**
- **The relationship between the amount of source charge (Q_0 in case 1, q_+ in Case 2) and $q_{induced}$ is the same!** This must be true because the dielectric must polarize the same way in response to a given amount of charge on the plates, regardless of whether there is a battery attached or not.

Figures from Mazur, *Principles and Practice of Physics* (to be published by Addison-Wesley).

<u>Effect of dielectric</u> w/fixed source charges

(handout) $\vec{E}$ reduced due to pol of dielectric (go through beginning 2/3

~~How much charge is induced?~~



$+Q_0$ $-Q_0$

$E_{plates} = \frac{\sigma_0}{\epsilon_0} = \frac{Q_0}{A\epsilon_0}$

Add dielectric: $\vec{E}$ reduced (draw in above)

$$\vec{E}_{total} = \vec{E}_{plates} + \vec{E}_{induced} = \frac{Q_{total}}{A\epsilon_0} = \frac{Q_0 + q_{induced}}{A\epsilon_0} \text{ to right}$$

$q_{induced}$ is $\ominus$ $\Rightarrow$ Q_{total} is less $\Rightarrow$ $\vec{E}_{total}$ is less

Define dielectric constant κ to quantify how much less

$$E_{total} = \frac{E_0}{\kappa} = \frac{Q_0}{\kappa A\epsilon_0}$$

How much induced charge? Combine these two results:

$$\frac{Q_0 + q_{induced}}{A\epsilon_0} = \frac{Q_0}{\kappa A\epsilon_0} \Rightarrow Q_0 + q_{induced} = \frac{Q_0}{\kappa} \cancel{\Rightarrow q_{ind} = Q_0\left(1 - \frac{1}{\kappa}\right)}$$

$$\Rightarrow q_{induced} = -Q_0\left(1 - \frac{1}{\kappa}\right)$$

11:45

$\boxed{\text{CT 1}}$ $\vec{E}_{tot}$, ΔV_{cap} both smaller

$\boxed{\text{CT 2}}$ C is <u>greater</u>: [Remember
$Q = C\Delta V_{cap}$ <u>defines</u> C $\Rightarrow$ $C \equiv \frac{Q}{\Delta V_{cap}}$

remind beforehand

So Q same, ΔV_{cap} smaller $\Rightarrow$ C greater

Specifically: $E_{total} = \frac{E_0}{\kappa}$

means $\Delta V_{cap} = E_{total}\, d = \frac{E_0 d}{\kappa} = \frac{\Delta V_{cap,0}}{\kappa}$ decreases by factor of κ

$\Rightarrow C = \frac{Q_0}{\Delta V_{cap}} = \frac{Q_0}{(\Delta V_{cap,0}/\kappa)} = \kappa C_0$ where C_0 = capacitance w/o dielectric

Capacitance increases. $\boxed{\text{Demo: capacitor w/cheese}}$

~~What about energy? ... increase?~~
~~What about energy? Here really important to think~~
~~Think about~~

What if instead we keep the battery connected?
With dielectric:

E_{total} must stay same b/c $\Delta V_{cap} = E_{total}\, d$ is true regardless of whether there is a dielectric

$\boxed{CT3}$ here

$\Rightarrow$ charge q_+ on ⊕ plate must increase!

$$E_{total} = \frac{q_+ + q_{induced}}{A\epsilon_0} = \frac{Q_0}{A\epsilon_0}$$

As $q_{induced}$ is ⊖, $q_+ + q_{ind} = Q_0$ tells us $q_+ > Q_0$

How much charge on ⊕ plate?

Still true that E_{total} is reduced from field of q_+ alone by κ:

$$E_{total} = \frac{E_{plates}}{\kappa} = \frac{q_+}{A\epsilon_0 \kappa}$$

Combine:

$$\frac{q_+ + q_{induced}}{A\epsilon_0} = \frac{q_+}{A\epsilon_0 \kappa} \Rightarrow q_{induced} = -q_+\left(1 - \frac{1}{\kappa}\right)$$

SAME AS BEFORE!

Substitute this into $q_+ + q_{induced} = Q_0$

$$\Rightarrow q_+ - q_+\left(1 - \frac{1}{\kappa}\right) = Q_0 \Rightarrow q_+ = \kappa Q_0$$

Charge goes up by factor of κ!

Consistent w/ capacitance going up by κ: $C = \frac{Q}{\Delta V_{cap}}$

A parallel plate capacitor is charged and disconnected from the battery used to charge it. Then, a slab of dielectric is inserted between the plates, completely filling the space between the plates.

Q_0 stays same!

How do the strength of the electric field between the plates, E_{total}, and the potential difference from one plate to another, ΔV_{cap}, compare to their values before the dielectric is added?

1. E_{total} and ΔV_{cap} are both larger.
2. E_{total} remains the same and ΔV_{cap} is larger.
3. E_{total} and ΔV_{cap} both remain the same.
4. E_{total} is smaller and ΔV_{cap} remains the same.
5. E_{total} and ΔV_{cap} are both smaller.
6. Need more information.

$\Delta V_{cap} = E_{total}\, d$

A parallel plate capacitor is charged and disconnected from the battery used to charge it. Then, a slab of dielectric is inserted between the plates, completely filling the space between the plates.

How does the capacitance of the capacitor C compare to its value without the dielectric?

1. C is smaller because ΔV_{cap} is smaller.
2. C remains the same because the plate shape and separation are the same.
3. C is greater because ΔV_{cap} is smaller.

Adding a dielectric to a capacitor increases C — increases amount of charge for a given ΔV_{cap}

$$\Delta V_{cap} = E_{total}\, d = \frac{E_0\, d}{K} = \frac{\Delta V_{cap,0}}{K}$$

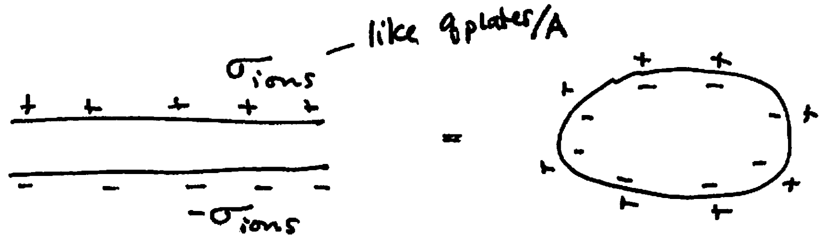
$$C = \frac{Q}{\Delta V_{cap}} = \frac{Q_0}{(\Delta V_{cap,0}/K)} = K\underbrace{\left(\frac{Q_0}{\Delta V_{cap,0}}\right)}_{\text{original } C_0} = K C_0$$

A parallel plate capacitor is charged and remains connected to the battery. Then, a slab of dielectric is inserted between the plates, completely filling the space between the plates.

ΔV_{cap} is fixed!

How do the strength of the electric field between the plates, E, and the charge on the positive plate, Q_+, compare to their values before the dielectric is added?

1. ~~E and Q_+ are both larger.~~
2. E remains the same and Q_+ is larger.
3. E and Q_+ both remain the same.
4. E is smaller and Q_+ remains the same.
5. E and Q_+ are both smaller.
6. Need more information.



Cell membrane: potential difference is fixed by the chemical reaction that the ion pump proteins use to transport charge across the membrane

$\Rightarrow$ if $\kappa = 8$ for the lipid material between the charge layers, then σ_{ions} increases to 8x what you would have without the lipid

Previously ~~we found that you did a problem~~ we found $\vec{E}$ in cell membrane using

$\Delta V_{in\ out} = +70\ mV$

Found $E = \dfrac{+70 \times 10^{-3}\ V}{7 \times 10^{-9}\ m} = 1.0 \times 10^{7}\ V/m = 1.0 \times 10^{7}\ N/C$

Is this still the right field? Yes: but the charge density of ions producing this field ~~has changed~~ is different.

$E_{total} = \dfrac{\sigma_{total}}{\epsilon_0}$ with $\sigma_{total} = \sigma_{ions} + \sigma_{induced}$

$\Rightarrow \sigma_{total} = \epsilon_0 E_{total}$ ~~and we just showed $\sigma_{ions} = \kappa \sigma_{total}$~~

~~$\Rightarrow E_{total} = \left(\dfrac{\sigma_{ions}}{\kappa}\right)\dfrac{1}{\epsilon_0}$~~ ~~$\Rightarrow \sigma_{ions} = \kappa \epsilon_0 E_{total}$~~

(can put in #s)

$= \left(8.85 \times 10^{-12}\ C^2/Nm^2\right)\left(1.0 \times 10^{7}\ N/C\right) = 8.85 \times 10^{-5}\ C/m^2$

We just showed $q_{+} = \kappa Q_0$ which means

$\sigma_{ions} = \kappa \sigma_{total}$ $\Rightarrow$ 8x as much charge if $\kappa = 8$!

The lipid enables cell to store much more charge!

[redacted]

A charged parallel plate capacitor remains connected to the battery and then a slab of dielectric is inserted between the plates. Compared to the energy before the slab was inserted, the energy stored in the capacitor with the dielectric, U_{cap}, is

ΔV_{cap} same, Q increases

1. less
2. the same
3. greater

$$U_{cap} = \frac{1}{2} Q \Delta V_{cap} \quad \text{greater}$$

$$= \frac{1}{2} \underbrace{C}_{\text{increases}} \Delta V_{cap}^2$$

$$= \frac{1}{2} \frac{Q^2}{C}$$

12:15 How does this affect how much energy is stored?

Energy in dielectrics handout:

for capacitors normally we have fixed ΔV_{cap}

$\rightarrow$ dielectric increases $C \Rightarrow$ increases U_{cap}

$$U_{cap} = \frac{1}{2} C \Delta V_{cap}^2$$

Most important biochemical effect: effect of dielectric solvent on ions

Significance of reduced $\vec{E}$ in dielectric — profoundly important for chemistry in water (i.e. biochemistry!).

Water has very high $\kappa \approx 80$

$\Rightarrow$ electric field of charged particles in water is dramatically reduced

(using $\frac{1}{4\pi\epsilon_0}$ instead of k)

$$\vec{E}_{Na^+} = \frac{+e}{4\pi\epsilon_0 r^2 \kappa}\hat{r}$$

κ ~80 for water

Na^+ $\quad$ Cl^-

Force on Cl^- is much weaker!

$\boxed{CT}$ potential energy of two charges (binding energy)

Binding energy becomes much smaller

Recall we calculated the potential energy change of two charges

$$U_{charges} = q_2 \Delta V_{\infty 2} = q_2\left(\frac{q_1}{4\pi\epsilon_0 r_{12}}\right) = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$

q_2: charge of 2^{nd}

$\Delta V_{\infty 2}$: potential difference from ∞ far away to final location of 2

$q_1 \xrightarrow{r_{12}} q_2$

Because $\Delta V_{\infty 2} = -\int_{\infty}^{r_{12}} \vec{E}_1 \circ d\vec{r}$, if $\vec{E}$ decreases by κ, so will ΔV!

$\Rightarrow$ in presence of dielectric, binding energy has factor of κ:

$$U_{binding} = \frac{q_1 q_2}{\kappa 4\pi\epsilon_0 r_{12}}$$

Na^+Cl^- ion pair: in air, $\kappa \cong 1$

$$U_{binding} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}\kappa} = \frac{(1.6\times10^{-19}C)(-1.6\times10^{-19}C)}{4\pi(8.85\times10^{-12}C^2/Nm^2)(0.24\times10^{-9}m)(1)} = 9.6\times10^{-19}J$$

Thermal energy at room $T = k_B T = (1.38\times10^{-23}J/K)(300K) = 4.1\times10^{-21}J$

Salt does not come apart in air at room T!

In water: $U_{binding} = \frac{9.6\times10^{-19}J}{80} = 1.2\times10^{-20}J$ only 3x thermal energy!

Two charged particles with charges +e and –e are surrounded by a dielectric ($\kappa > 1$). How does the electric potential energy of the charges $|U_{charges}|$ compare to the energy of the same arrangement in air or vacuum ($\kappa = 1$)?

1. $|U_{charges}|$ is smaller.
2. $|U_{charges}|$ remains the same.
3. $|U_{charges}|$ is greater.
4. Need more information.

Energy stored in dielectrics

Energy stored in capacitors:

All the relationships worked out for the energy stored in a charged capacitor still apply, because nothing about the derivation depended on there being no dielectric. Just keep in mind that the capacitance now has a factor of the dielectric constant in it. Using q_+ for the charge on the positive capacitor plate these relationships are:

$$U_{cap} = \frac{1}{2}C\left(\Delta V_{cap}\right)^2 = \frac{1}{2}\frac{q_+^2}{C} = \frac{1}{2}q_+\Delta V_{cap} \qquad \text{with } C = \frac{\kappa A\varepsilon_0}{d}$$

Electric potential energy of arrangements of charged particles:

Charged particles represent a case in which the source charges of the electric field are fixed. (The particles have the same charge whether there is a dielectric or not!) The electric field of fixed source charges is reduced by a factor of κ in the presence of a dielectric, so the electric potential energy change associated with moving through that electric field also decreases by a factor of κ:

$$E_{total} = \frac{E_0}{\kappa} \text{ (fixed source charges) and } \Delta U_{AB} = q\Delta V_{AB} = -q\int_A^B \vec{E}_{total} \bullet d\vec{r}$$

gives $\Delta U_{AB,diel} = \frac{\Delta U_{AB}}{\kappa}$ (fixed source charges).

Likewise the total electric potential energy of an arrangement of charges also decreases by a factor of κ:

$$U_{charges,diel} = \frac{U_{charges,0}}{\kappa} = \left(\frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}\right)\frac{1}{\kappa}$$ for the case of two charges q_1 and q_2 separated by r_{12}

Energy density of the electric field:

The energy density of the electric field **increases** by a factor of κ when a dielectric is present. To show this, we determine how the energy density changes with the addition of a dielectric at fixed electric field strength, and hence fixed potential difference, so that we compare the energy density with the same values of electric field E_0.

For fixed potential difference, the total energy in the capacitor with the dielectric is

$$U_{cap} = \frac{1}{2}C\left(\Delta V_{cap,0}\right)^2 = \frac{1}{2}C\left(E_0 d\right)^2 = \frac{1}{2}\frac{\kappa A\varepsilon_0}{d}E_0^2 d^2 = \frac{1}{2}\kappa\varepsilon_0 E_0^2 Ad$$

and the last expression is just the energy density multiplied by the volume in the capacitor:

$$U_{cap} = \frac{1}{2}\kappa\varepsilon_0 E_0^2 Ad = u_E(volume)$$

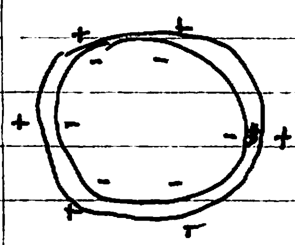
so, the energy density in an electric field in a dielectric is

$$u_E = \frac{1}{2}\kappa\varepsilon_0 E_0^2$$

These results apply to any situation, even when there is no dielectric! Just remember that for air and vacuum, $\kappa=1$ (for vacuum, $\kappa=1$ exactly; for air $\kappa=1.0006$, but for our purposes you can just use $\kappa=1$.) Think of κ as telling you how much the dielectric polarizes. In vacuum there is nothing there to polarize so we just have $\kappa=1$.

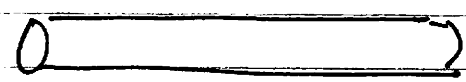
How ~~the~~ are electrical signals we measure outside the body generated?
- electrocardiogram: stick electrodes onto skin, measure potential differences → infer heart activity
- EEG ⇒ brain activity

~~[illegible]~~ Impossible to understand for ~~[illegible]~~ fully charged cells OR fully discharged cells



$\vec{E}$ is zero outside!

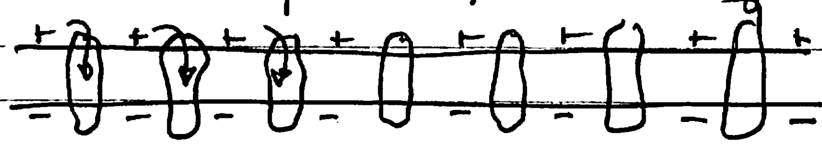
Also true of long cylindrical cells such as neurons



(If discharged, obviously $E = 0$ everywhere)

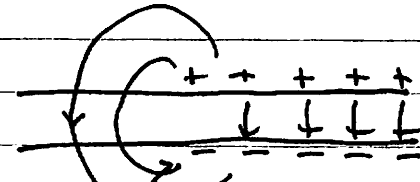
So where do electrical signals we can measure come from?
Come from process of discharging

Consider cell as parallel flat sheets again



Fully charged cell membrane ("polarized")
When a muscle cell contracts or a nerve signal initiates, channels open up ~~[illegible]~~ inside the membrane to let charge travel in: "Depolarization"
Heart is made up of many of these cells next to each other.
Doesn't happen everywhere at once — starts at one end of heart and travels along

~~What does~~ ⇒ partly charged capacitor



What does $\vec{E}$ ~~look~~ like @ ends? a dipole
NOT zero outside (though much weaker than the field inside!) — Measure this field!