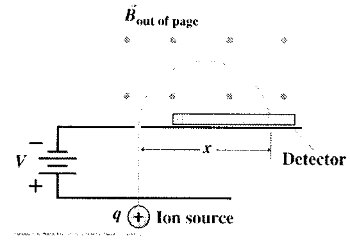
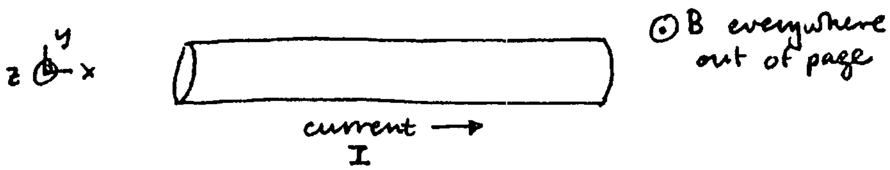
Problems/questions for class, 3/25/10

A mass spectrometer is used to identify the charge-to-mass ratio of charged particles in the apparatus shown. The accelerating voltage is 10 kV. Protons used to calibrate the instrument land on the detector 10 cm from the entrance slit. What is the magnetic field strength?



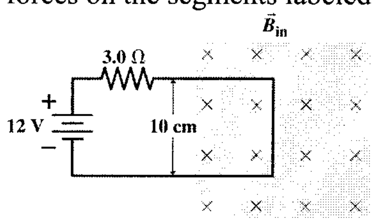
An unknown ion reaches the detector twice as far from the entrance point as a proton used for calibration What do we know about the unknown ion?

If current corresponds to negatively charged electrons moving, what is the direction of the force on the wire?

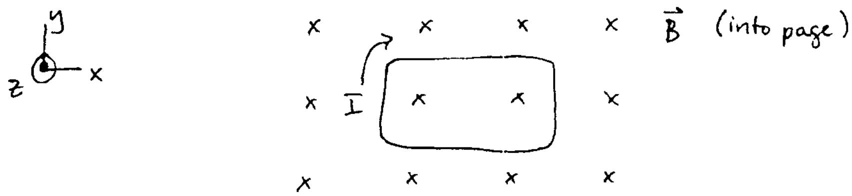


You wonder if a clothesline in the Earth's magnetic field could be supported by running an electric current through it. For simplicity consider a clothesline in Costa Rica, near the equator, where the magnetic field is horizontal and points from south to north, and the field strength is approximately 50 μT. Assume the clothesline is 10 m long and when loaded with wet clothes it weighs 5.0 kg. Can this be done? What current would be required and which direction would the clothesline need to be oriented?

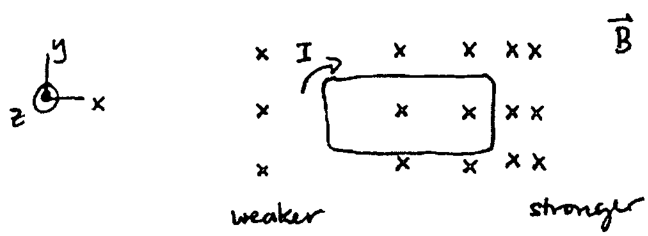
Find the *direction* of the net magnetic force on the circuit shown by finding the directions of the forces on the segments labeled I, II, and III, and considering how these three forces will add.



The magnetic field below is uniform. What is the direction of the net magnetic force on the rectangular current loop shown? (Find the forces on each of the four sides of the loop and add them up.)



The **nonuniform** magnetic field shown increases in strength from left to right. What is the direction of the net magnetic force on the rectangular current loop?



Announcements 3/25/10 (go over at end)

Change to prelab next week: Monday and Tuesday lab members postpone calculating the field of the coils until after class Tuesday.

Friday afternoon office hours this week and next: 1:30 – 3 (no junior lab)

Reading details: For today and next Tuesday, read after class because our approach is quite different from Wolfson.
In 26.4, skip Hall effect.
We are not going to cover the Biot-Savart Law (beginning of 26.5) quantitatively, just understand it qualitatively.
We will use the **results** of examples 26.4 and 26.5 but you do not need to be able to derive them.
We will cover the part on "Magnetic force between conductors" at the end of section 26.5.

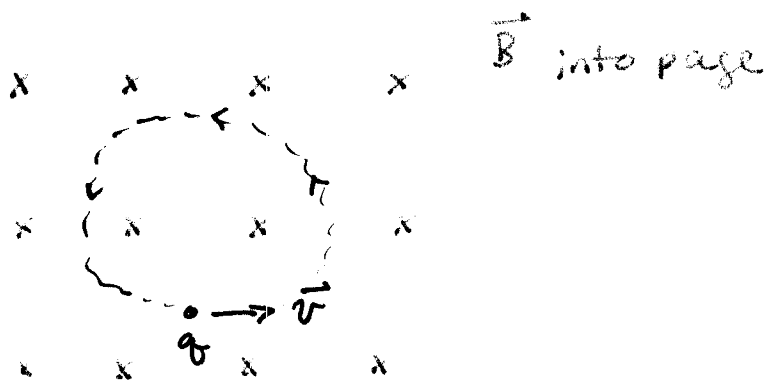
Feedback:
General purpose: Feedback allows me to hear from everyone in the class and to adjust a few aspects of how I am teaching the class to best match what I hear. With such a large class, there's a broad range!
Most important element: what *you* do

Key ideas from last time

Magnetic fields $\vec{B}$ fill the space around Earth and magnets
Magnetic fields exert forces on other magnets
Surprisingly: although $\vec{B}$ does not exert force on *stationary* charges, magnetic forces act on *moving* charges

$$\vec{F}_{by\,\vec{B}\,on\,q} = q\vec{v} \times \vec{B}$$

In uniform $\vec{B}$ perpendicular to motion, charged particles go in circular orbits with radius $r = \dfrac{mv}{qB}$



J.J.

3/25/2010

Today

(1) Circular orbits: mass spectrometer example
(2) Magnetic forces on currents: $\vec{F} = I\vec{L}\times\vec{B}$
[illegible] - issue for pacemakers in MRI!
(3) Sources of magnetic fields: <u>currents</u>
- $\vec{B}$ of long straight wire
- force one long straight wire exerts on another parallel wire

<u>Mass spectrometer</u>: uses circular orbits to separate chemical species by charge to mass ratio q/m
- ionize molecules ~~[illegible]~~ $\Rightarrow$ charged ~~[illegible]~~ fragments
- accelerate fragments through V in region w/no $\vec{B}$

$$\Delta K = -\Delta U^E \quad \text{so} \quad \frac{1}{2}mv^2 = qV$$

$$\Rightarrow v = \sqrt{\frac{2qV}{m}}$$

- after acceleration ions leave $\vec{E}$, go into region w/ $\vec{B}$ and follow <u>circular orbits</u> <u>according</u> to this v

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

- land on detector a distance $x = 2r$ from entrance which <u>depends on q/m</u>
- determine charge q separately $\Rightarrow$ find mass (why it's called "mass spectrometer")

If know where protons land:
something that lands twice as far away must have $4\times\ \frac{m}{q} \Rightarrow \frac{1}{4}\ \frac{q}{m}$

double this $\longrightarrow$ $x = \frac{2}{B}\sqrt{\frac{2mV}{q}}$ $\Leftarrow$ double this w/ V, B constant

Problem:

Find B given values

Principle: use result from before, solve for B

$$x = 2r = \frac{2}{B}\sqrt{\frac{2mV_{accel}}{q}} \Rightarrow B = \frac{2}{x}\sqrt{\frac{2mV_{accel}}{q}}$$

subst: use SI units (meters, kg, Coulombs, Volts)

$\Rightarrow$ B in Tesla

$x = 0.10$ m

$V_{accel} = 10$ kV $= 10 \times 10^{3}$ V

$m = 1.67 \times 10^{-27}$ kg

$q = 1.6 \times 10^{-19}$ C

$\Rightarrow B = 0.20$ T

Units are really not helpful here b/c rel'nship between B units & other units is not clear!

A mass spectrometer is used to identify the charge-to-mass ratio of charged particles in the apparatus shown. The accelerating voltage is 10 kV. Protons used to calibrate the instrument land on the detector 10 cm from the entrance slit. What is the magnetic field strength?

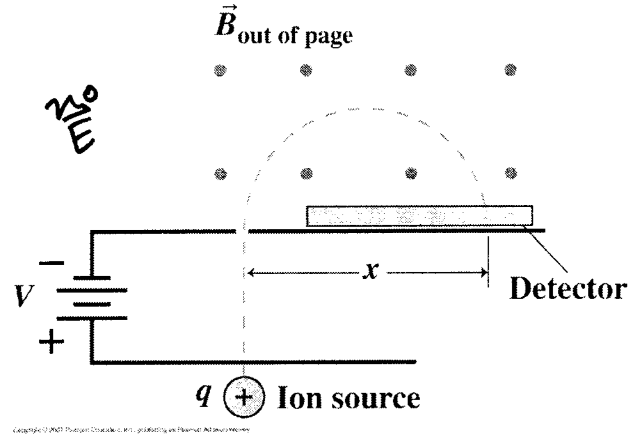
Accelerate charged fragments through V

$\Delta K + \Delta U^E = 0$

$\frac{1}{2}mv^2 - qV = 0$

$v = \sqrt{\frac{2qV}{m}}$

no $\vec{E}$



After going through slit,

no $\vec{E}$ ⇒ v const ⇒ circ orbits of $r = \frac{mv}{qB}$

Sub in for v

⇒ $r = \frac{1}{B}\sqrt{\frac{2mV}{q}}$ depends on V, B, and q/m

Land on detector distance $x = 2r$ from entrance slit

(Separate measurement gives q ⇒ allows determining m "mass spectrometer")

Protons land at $x = 0.10$ m — what is B?

Use $x = 2r = \frac{2}{B}\sqrt{\frac{2mV}{q}}$ sub in for m, q (proton) V (given in problem)

Using all SI units → B in Tesla

x in meters, m in kg, q in C, V in volts

- If there is enough room for particles to go in a complete circle, will orbit repeatedly

Time for one orbit:

$$T = \frac{\text{circumference}}{\text{speed}} = \frac{2\pi r}{v} = \frac{2\pi}{v}\left(\frac{mv}{qB}\right) = \frac{2\pi m}{qB}$$

Turns out that ~~accelerating~~ circling charges give off light waves — the electric and magnetic fields of these circling charges constitute a light wave!

We'll explain this in a few weeks

In the meantime: frequency of orbit = freq of light

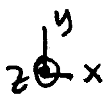
$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

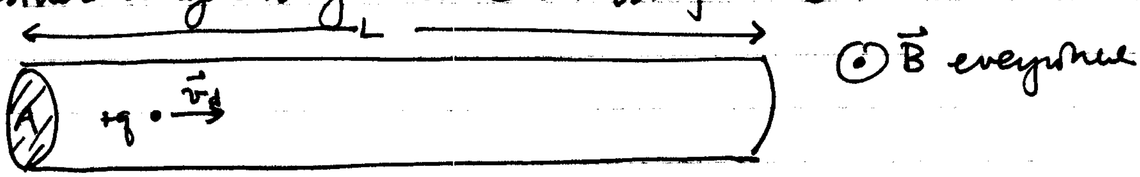
- This phenomenon is used in microwave ovens — circling e^- in magnetic field generate the microwaves. HW problem about this! Also origin of northern lights (aurora borealis)!

Magnetic force on current

Current = lots of identical charged particles w/ average velocity in same direction

Consider long straight wire in uniform $\vec{B}$:





current $I = nAqv_d$ in direction of $\vec{v}_d$

cross-sect area

mobile charges/vol

Net force = sum of all forces on all moving charges!

$$\vec{F}_{net} = Nq\vec{v}_d \times \vec{B} = n\underbrace{AL}_{\text{volume of wire}}\, q\vec{v}_d \times \vec{B} = nA\left(Lq\vec{v}_d\right)\times\vec{B}$$

(N = total # of moving charges)

Define vector $\vec{L}$ to point in direction of current = direction of $q\vec{v}_d$ (⊕ charges move ~~…~~ in direction of current
⊖ charges move opposite current)

Then can rewrite $Lq\vec{v}_d$ as $qv_d\vec{L}$ just ~~…~~ assigning the ~~…~~ direction to $\vec{L}$

(for the case above:

$$\left.\begin{array}{l}\vec{L} = L\hat{\imath}\\ q\vec{v}_d = qv_d\hat{\imath}\end{array}\right\} \text{ so } Lqv_d\hat{\imath} \text{ is either } \left\{\begin{array}{l}Lq\vec{v}_d\\ \text{or}\\ qv_d\vec{L}\end{array}\right\}$$

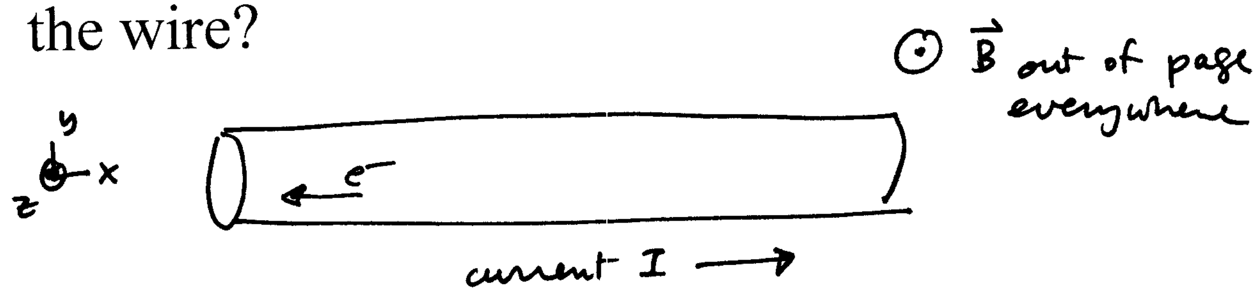
$$\Rightarrow \vec{F}_{net} = nAqv_d\vec{L}\times\vec{B} = I\vec{L}\times\vec{B} \quad \text{for straight wire in uniform } \vec{B}$$

Says force is ⊥ to both $\vec{B}$ and direction of current
Strength depends on amount of current, length of wire, field, & angle

$\boxed{\text{CT}}$

11:50

If current is carried by the motion of negative electrons, what is the direction of the force on the wire?



$I\vec{L}\times\vec{B}$ points down with

L to right

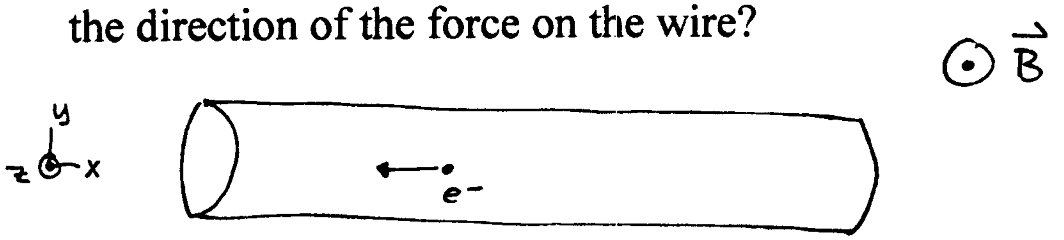
$\vec{B}$ out

1. +y
2. -y

$$q\vec{v} = -e(v \text{ to the left})$$

$$-e(v(-\hat{i})) = ev\hat{i}$$

If current is carried by the motion of negative electrons, what is the direction of the force on the wire?



current flows to the right

$I \longrightarrow$

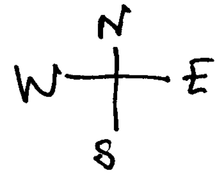
1. +y (up)
2. -y (down)

The force is down regardless of whether the current is carried by negative charge moving left or positive charge moving right. In either case the current points to the right and so does $\vec{L}$ in $I\vec{L}\times\vec{B}$, so the direction of the force is the same. When we derived $I\vec{L}\times\vec{B}$ by considering the force on all the moving charges in the wire, we can substitute $I\vec{L}$ for $nqAL\vec{v}_d$ whether q is positive or negative because if q is negative, $\vec{v}_d$ reverses direction and thus the two changes of sign cancel out.

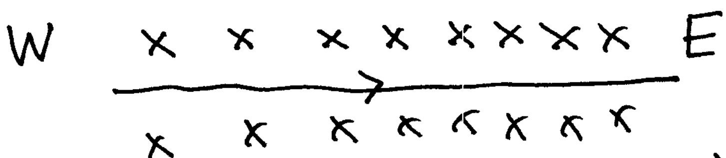
You wonder if a clothesline in the Earth's magnetic field could be supported by running an electric current through it. For simplicity consider a clothesline in Costa Rica, near the equator, where the magnetic field is horizontal and points from south to north, and the field strength is approximately 50 μT. Assume the clothesline is 10 m long and when loaded with wet clothes it weighs 5.0 kg. Can this be done? What current would be required and which direction would the clothesline need to be oriented?

Principle: $\vec{B}$ to exert force up to oppose F_g down $= mg$

Need to find amount & direction of current to give appropriate $\vec{F}_B$ up



$\vec{B}$ points from South to north



Direction? (Want $I\vec{L}\times\vec{B}$ up) $\boxed{\text{current to right}}$

$I\vec{L} \perp \vec{B} \Rightarrow F_B = ILB = mg$

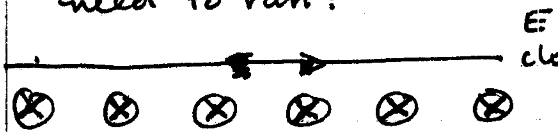
$$I = \frac{mg}{LB} \qquad \left.\begin{array}{l} m = \cancel{8}5.0\ \text{kg} \\ g = 10\ \text{m/s}^2 \\ L = 10\ \text{m} \\ B = 50\times10^{-6}\ \text{T} \end{array}\right\} \Rightarrow I = 100\times10^{3}\ \text{A}$$

Clothesline Problem

Principle: $\vec{B}$ exerts force on current that could balance $\vec{F}_g$
Goal: amount of current & direction so that $\vec{F}_B + \vec{F}_g = 0$

Need $\vec{F}_B$ up
$\vec{B}$ points from ~~north~~ south to ~~south~~ north so which way does current need to run?

W



clothesline runs east-west

N ~~out of~~ into pg
S ~~into~~ out of pg

N
W + E
S

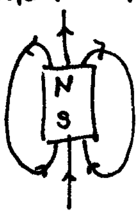
Want $I\vec{L}\times\vec{B}$ up → requires current left or right? ~~left~~ right
(corresponds to ~~west~~ east)

Want $F_B = ILB$ to balance $F_g = mg$

$$\Rightarrow \quad ILB = mg \Rightarrow I = \frac{mg}{LB} = \frac{(5.0\text{ kg})(10\text{ m/s}^2)}{(10\text{ m})(50\times10^{-6}\text{ T})} = 100\text{ kA!!}$$

(not practical!) (though could dry clothes this way...)

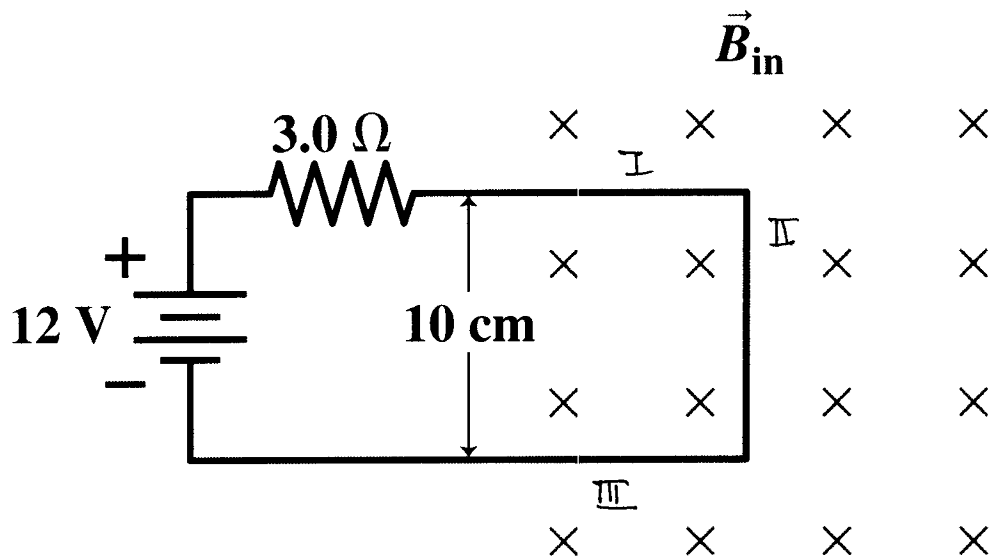
Note to self:



Earth's N pole is a S mag pole

Magnetic levitation of trains etc:
much bigger $\vec{B}$ by using magnets to produce $\vec{B}$
then currents still need to be big also but not as big....

Find the *direction* of the net magnetic force on the circuit shown by finding the directions of the forces on the segments labeled I, II, and III, and considering how these three forces will add.



The force is to the right. The force on segment I is up; the force on segment III is down. These two segments have the same length and the same current, so because the magnetic field is uniform, the forces on I and III cancel. Therefore the net force on the circuit is the force on II, which is to the right. All three directions come from the right-hand rule for cross products and $\vec{F} = I\vec{L} \times \vec{B}$ with $\vec{L}$ in the direction of the current.

12:05 Why do people with pacemakers need to stay away from strong $\vec{B}$ such as in MRI machines?

Not b/c pacemakers are made with materials that can be picked up with a magnet – but b/c of currents – $\vec{B}$ exerts force on current

(Skip problem about circuit $\S$ partly in $\vec{B}$ @ bottom of 1st side of problem sheet)

Consider force on a circuit in a uniform $\vec{B}$

Represent circuit as a rectangular loop of current

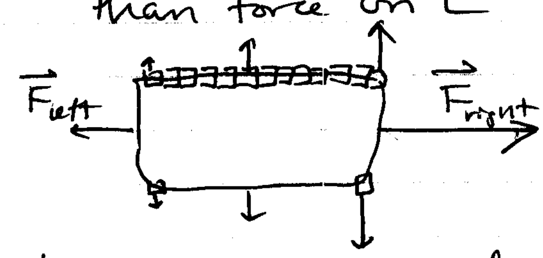
$\boxed{\text{CT}}$ Direction of net force? zero – top & bottom cancel left & right cancel

Any loop → zero net force in uniform $\vec{B}$ (can show)

What if $\vec{B}$ is not uniform but changes L to R?

$\boxed{\text{CT}}$ Direction of net force is to R

$\vec{B}$ stronger @ R → force on R stronger than force on L → net force is to right

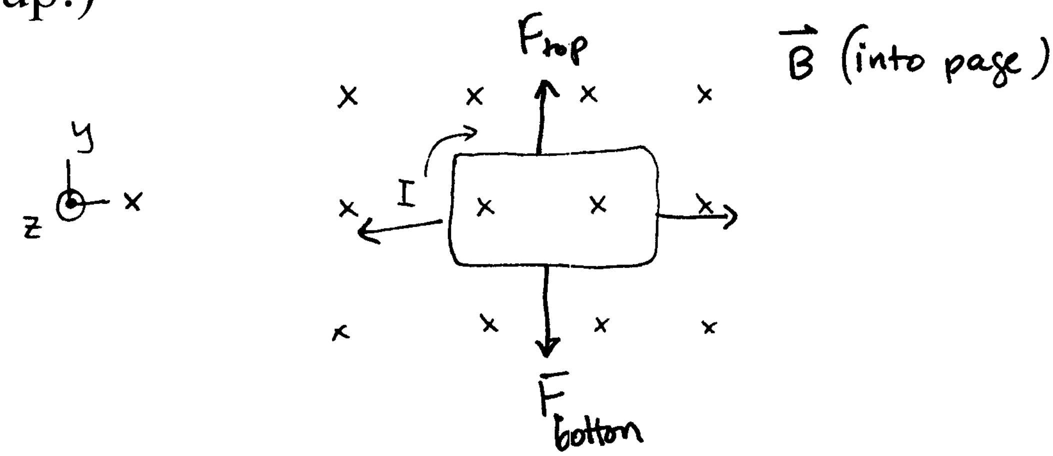


Top & bottom still cancel:

- think of dividing top & bottom into segments
- force on each segment varies w/strength of $\vec{B}$ but always cancel top & bottom

(To find force would integrate $\vec{F} = \int I d\vec{l} \times \vec{B}$)

The magnetic field shown below is uniform. What is the direction of the net magnetic force on the rectangular current loop shown below? (Find the forces on the four sides and add them up.)



1. +x
2. –x
3. +y
4. –y
5. The net force is zero.
6. Need more information.

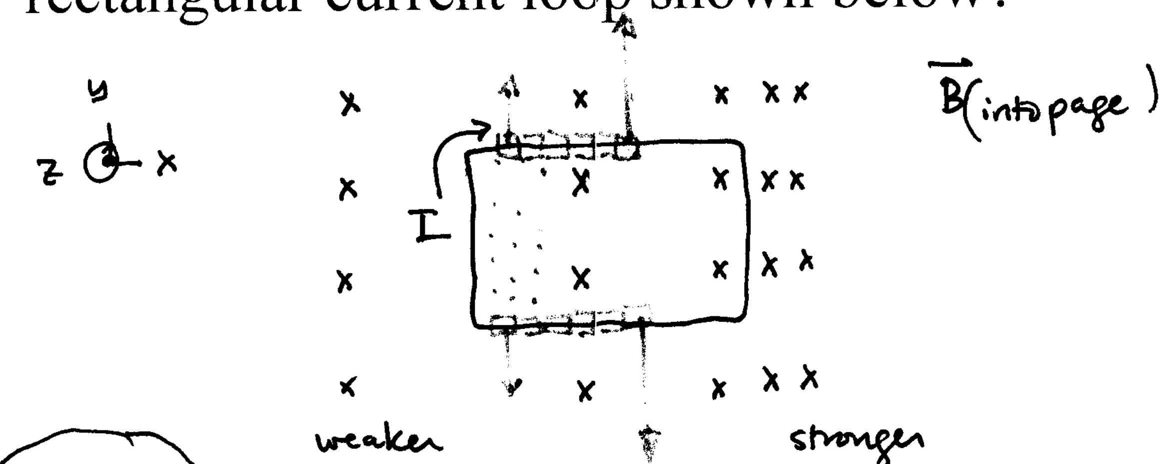
Forces on top & bottom add to zero

Forces on left & right add to zero

$\rightarrow$ net $F = 0$

Any current loop in uniform B $\Rightarrow$ no net force

The **nonuniform** magnetic field shown below increases in strength from left to right. What is the direction of the net magnetic force on the rectangular current loop shown below?



1. +x right
2. –x
3. +y
4. –y
5. The net force is zero.
6. Need more information.

vertical forces still cancel

force on R is stronger than force on L b/c $\vec{B}$ stronger

$\Rightarrow$ points to R

$\vec{F}_{Right} + \vec{F}_{left} \Rightarrow$ to R

In nonuniform $\vec{B}$ $\rightarrow$ force on loop of current

What are the sources of $\vec{B}$?

Just as for $\vec{E}$, charges feel electric forces and are source of (produce) electric fields; for $\vec{B}$, moving charges feel magnetic forces AND are source of magnetic fields!

	ELECTRIC FIELD	MAGNETIC FIELD
FEEL FORCE	charges	moving charges (currents) (and magnets)*
SOURCE	charges	moving charges (currents) (and magnets)*
[FIELD LINES	start/end on charge	loops around current]

But similarity ends at shape of field:
$\vec{E}$ starts/ends on charge
$\vec{B}$ forms loops around current!

* Have atomic currents inside them!

Show demo: $\vec{B}$ of wire
(remember: compass needles turn so point along $\vec{B}$)
Show Mazur Fig 32.2

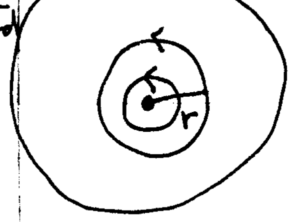
Direction of $\vec{B}$ loops:
put thumb in direction of current
fingers curl in direction of $\vec{B}$

Strength of $\vec{B}$: decreases with distance from wire

$$B(r) = \frac{\mu_0 I}{2\pi r}$$

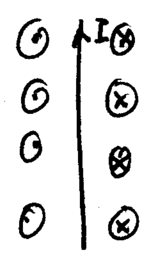
$\mu_0 \equiv$ "permittivity constant" — look up — units give B in Tesla if I is in A, r in m
$r = \perp$ distance from wire

Top view (wire toward you)



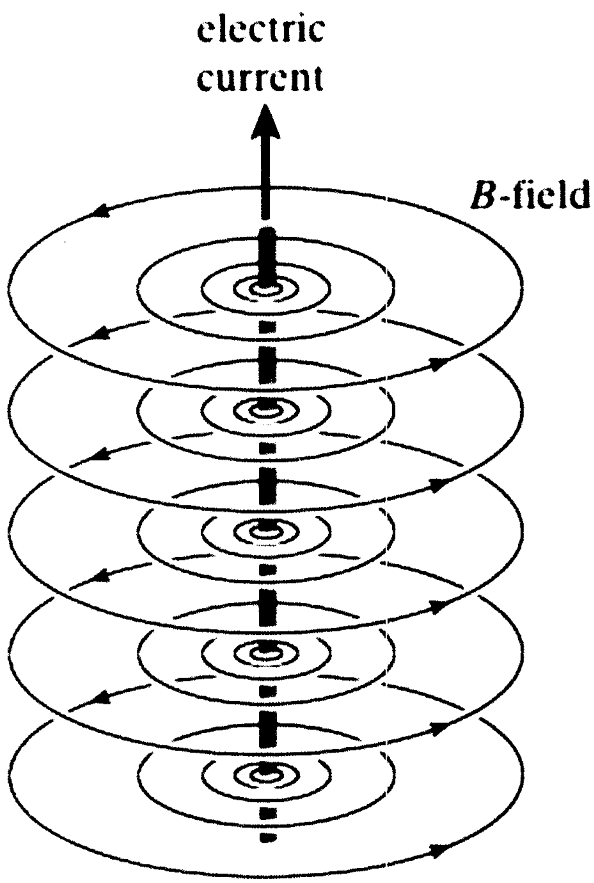
I out of page

Side view:

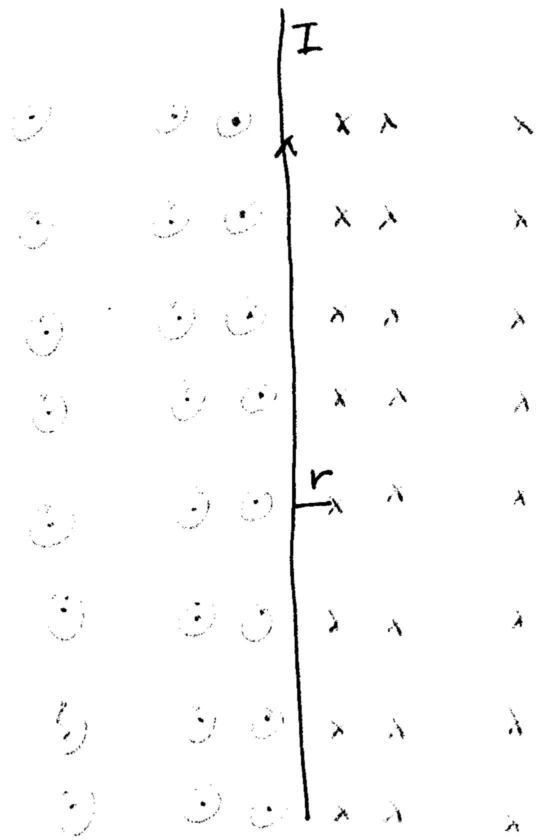


3D view

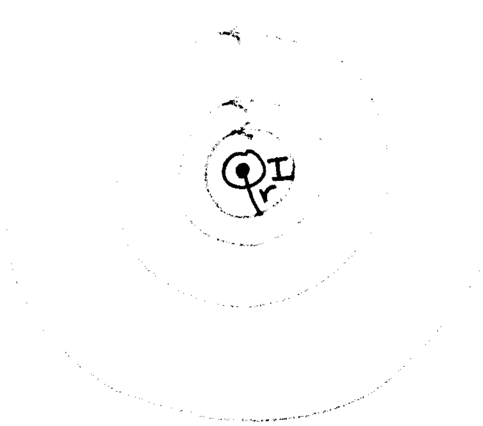
Side View:



(a)



Top view (current coming toward you):



Field strength: constant, look up in book covers

$$B(r) = \frac{\mu_0 I}{2\pi r}$$

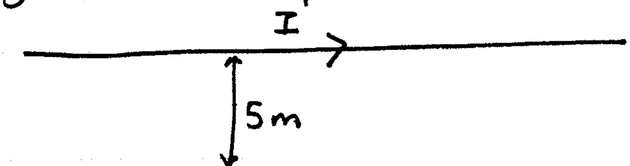
⊥ distance from wire

greater B from greater I

B decreases as get farther away

Use SI units $\rightarrow \vec{B}$ in Tesla

What is strength of $\vec{B}$ 5 m below a power line carrying 500 A?



$r = 5\,m$

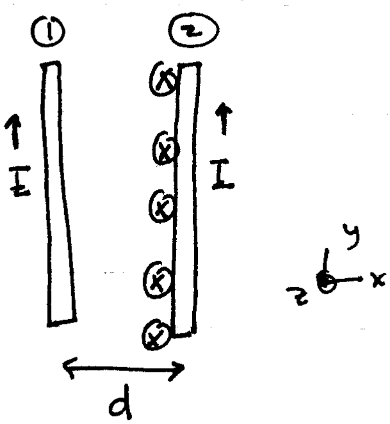
$I = 500\,A$

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(1.26\times10^{-6}\ N/A^2)(500\ A)}{2\pi(5.0\ m)} = 2.0\times10^{-5}\ T = 20\,\mu T$$

about 1/3 of Earth's field

[Long straight wire is one of three arrangements of current we'll find $\vec{B}$ of.]

12:25 What if we have two parallel wires carrying current – what force does the field of one exert on the current in the other?



$\boxed{CT}$ Direction of $\vec{F}_{by\,1\,on\,2}$?

Use $\vec{F}_{by\,1\,on\,2} = I_2\vec{L}_2\times\vec{B}_1$

field exerting force

current feeling force

Direction of $\vec{B}_1$ at location of wire 2:

RH rule says $\vec{B}_1$ is into page at wire 2

$\Rightarrow I_2\vec{L}_2\times\vec{B}_1$ is to left

Direction of $\vec{F}_{by\,2\,on\,1}$? must be opposite – if 1 pulls 2 to left, 2 must pull 1 to right. Attractive force!

Calculate strength of force: $\vec{L}_2 \perp \vec{B}_1$ so ~~~~ magnitude is just $I_2L_2B_1$

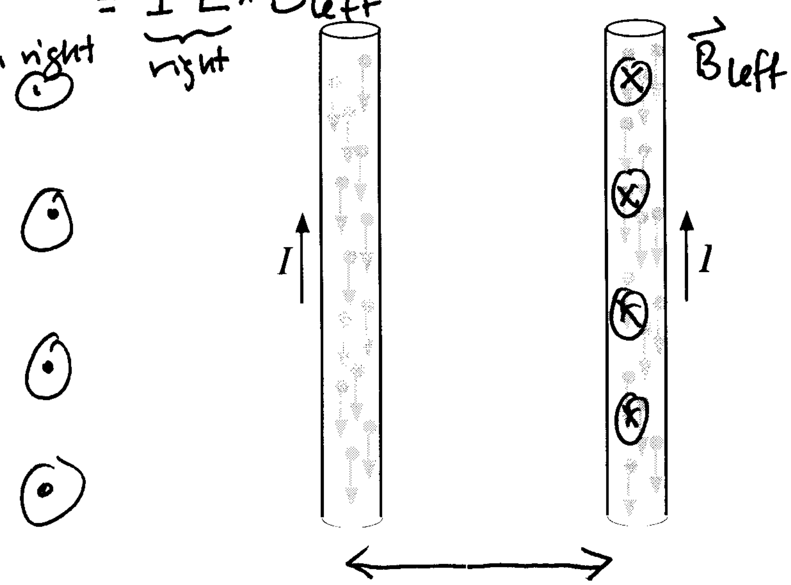
$$\vec{F}_{by\,1\,on\,2} = I_2\vec{L}_2\times\vec{B}_1 = I_2L_2\left(\frac{\mu_0 I_1}{2\pi r_{12}}\right)(-\hat{\imath})$$ direction from RH rule

r_{12} = distance d between wires

$$\Rightarrow \vec{F}_{by\,1\,on\,2} = \frac{\mu_0 I_1 I_2 L_2}{2\pi d}(-\hat{\imath})$$

Two parallel wires carry identical upward currents. The current is carried by electrons moving down. What is the direction of the magnetic force exerted by the left wire on the right wire?

$$\vec{F}_{\text{by left on right}} = \underbrace{I\vec{L}}_{\text{right}} \times \vec{B}_{left}$$



$r = d$ between wires

1. To the left.
2. ~~To the right.~~
3. Up.
4. Down.
5. The force is zero because the currents are parallel so the cross product is zero.

What if we reverse one of the currents?
Reverses either $\vec{L}_2$ or $\vec{B}_1$ ⇒ force becomes opposite direction → repulsive force

Key point is that if you have two parallel wires carrying current, such as in a circuit, exert forces on each other — if currents are small, forces are small, but large currents → significant forces!

Long straight ~~wire~~ current obviously has to be part of a bigger loop in order to carry current — so always an approximation — can use this expression for B when distance r is small compared to length of wire and when not near the ends.
(Force between two long ∥ wires is approximate — assumes wire is long enough that the fact that $\vec{B}$ is not exactly $B = \frac{\mu_0 I}{2\pi r}$ near the ends is not a problem.)

Biot-Savart Law

We are not using the Biot-Savart Law quantitatively.
What we will do next time is use it qualitatively — the Biot-Savart law says that if you don't have a long straight ~~wire~~ current, but instead have any shape, you can break that current up into straight segments
Each segment has a field that forms RH circles around it, and has magnitude $\frac{\mu_0 I}{4\pi r^2} dl$

strength increases ~~[illegible]~~ w/ current increasing
strength decreases w/ current decreasing

Total $\vec{B}$ comes from adding up fields of segments