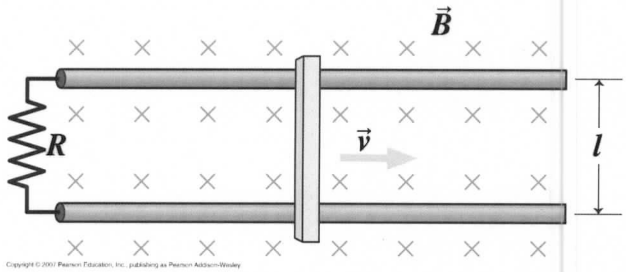
Announcements 4/8/10

Midterm tonight is at 7:30 in SC 101, details are on the information sheet online

Jake's question about how magnetite gets magnetized:
• turns out that the crystals in bacteria (50 nm diameter) are small enough ("single domain") that they do not need an external field to magnetize them — internal interactions are strong enough to align all the spins
• It appears (I'm not completely sure) that bigger pieces of magnetite do not spontaneously magnetize, but can be magnetized by relatively weak fields (i.e. Earth's field) especially when hot

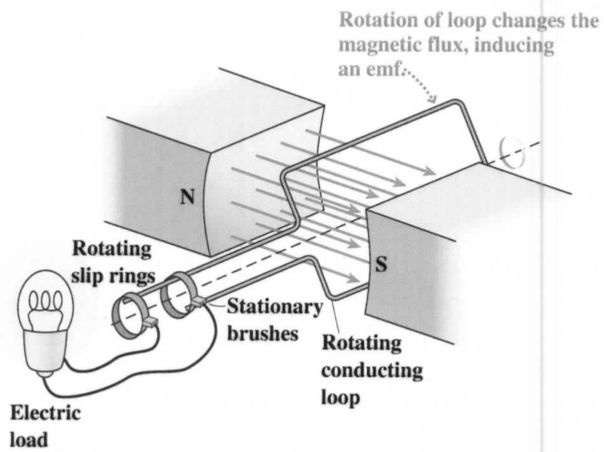
Problems and questions for class 4/8/10

A pair of parallel conducting rails a distance l apart are connected by a resistor R at the left. A conducting bar can slide across the rails with negligible friction while keeping electrical contact. The whole system is in a uniform magnetic field B as shown. You pull the bar at constant velocity v to the right. (a) What is the direction of the current in the resistor? (b) At what rate do you do work in pulling the bar? Where does this energy go?



For the "jumping ring" demonstration, find the induced current in the ring if the magnetic field strength changes at 120 T/s, the maximum magnetic field strength is 1.0 T, the diameter of the loop is 10 cm, and the resistance of the ring is 0.01 Ω.

An electric generator such as would be found in a hybrid car consists of a rectangular coil 20 cm by 30 cm, with 1000 turns. The coil sits in a uniform magnetic field of 0.050 T (produced by permanent magnets). When the car begins to slow down from highway speeds, the loop is turned at 10 rotations/second by the wheels of the car. What is the peak value of the induced emf in the coil?



Key ideas from last time

Induction: Current flows in a conducting loop if the magnetic flux through the loop is changing

Flux: $\Phi_B = \vec{B} \cdot \vec{A}$ in uniform field $= \int \vec{B} \cdot d\vec{A}$ in nonuniform

amount of magnetic field passing through the surface whose boundary is the loop

like counting field lines passing through the loop

for a coil with multiple turns: $\Phi_B = N \int \vec{B} \cdot d\vec{A} = N \vec{B} \cdot \vec{A}$

Induced emf is given by $\mathcal{E}_{ind} = -\dfrac{d\Phi_B}{dt}$ with $I_{ind} = \dfrac{\mathcal{E}_{ind}}{R_{loop}}$

Change in flux/induction can be caused by:

moving loop in limited region of $\vec{B}$ or nonuniform $\vec{B}$

loop with changing area or orientation in uniform $\vec{B}$

stationary loop with moving source of $\vec{B}$

source of $\vec{B}$ that changes its strength with time

Explain the first one in terms of magnetic forces on moving charges: "motional emf"

Lenz's law

Induced current flows in the direction that opposes the change in flux that caused it

specifically:

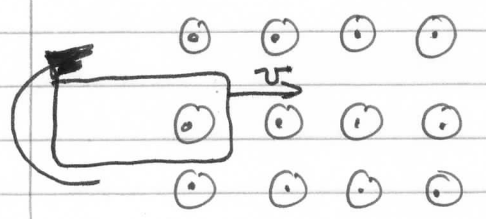
induced current I_{ind} produces a magnetic field $\vec{B}_{ind}$

the flux of $\vec{B}_{ind}$ partially offsets the change in Φ_B from the original field $\vec{B}$

- if Φ_B is increasing, $\vec{B}_{ind}$ is <u>opposite</u> original $\vec{B}$ ($\vec{B}_{ext}$ or just $\vec{B}$)

we'll use $\vec{B}_{ind}$ always for induced $\vec{B}$

Example: loop moving into field



Choose $\vec{A}_{loop}$ to be out of page

then as we showed last time

$$\Phi_B = \vec{B}\circ\underbrace{\vec{A}(t)}_{\text{amount of area w/ } \vec{B} \text{ going through it}} = BA(t) \quad \text{b/c } \vec{A}, \vec{B} \parallel$$

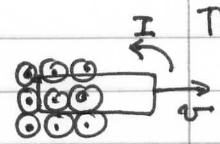
Induced current goes CW

$\vec{B}_{ind}$ <u>inside loop</u> is <u>into</u> page — opposite direction $\Rightarrow$ offsets $\vec{B}_{ext}$

$\boxed{\text{if you want to think mathematically:}}$ flux of $\vec{B}_{ind}$ is <u>negative</u>: $\vec{B}_{ind}\circ\vec{A}(t) = B_{ind}A(t)\underbrace{\cos 180°}_{-1}$

so offsets increasing flux

> NOTE: We calculate
>
> $$\mathcal{E} = -\frac{d\Phi_B}{dt}$$
>
> <u>only</u> using Φ_B from original (external) $\vec{B}$ — <u>not</u> using flux of B_{ind}



Then if loop is moving out so that $\Phi_B = \vec{B}\circ\vec{A}(t)$ is <u>decreasing</u>, current flows other way and $\vec{B}_{ind}$ points out of page, offsetting decreasing Φ_B

So we normally compute induced emf

$\mathcal{E} = -\frac{d\Phi_B}{dt}$ by finding abs values and then using Lenz's Law to find direction of current.

What about Φ_B for nonuniform fields?

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

still is a measure of the amount of field passing through the loop

think of dividing up surface into patches with area $d\vec{A}$, finding $d\Phi_B$ through each

$$d\Phi_B = \vec{B} \cdot d\vec{A}$$

and then adding them up by integrating

We'll do the one useful quantitative example of this in class Tuesday - flux of the field of a wire through a rectangular loop

However, we will think qualitatively about flux of nonuniform fields often - ask yourself if the amount of $\vec{B}$ passing through the surface is increasing or decreasing - is more of the surface occupied by stronger field?

Example: $\boxed{\text{CT}}$ pushing rectangular loop toward wire

direction of induced current?

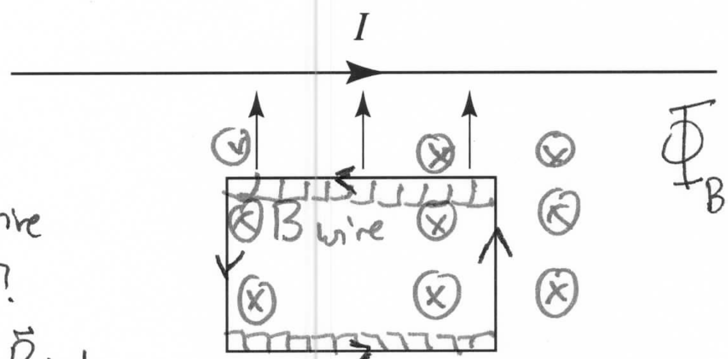
remember:

identify direction of original $\vec{B}$ ($\vec{B}_{wire}$ in this case) <u>inside</u> loop

is flux increasing or decreasing?

$\vec{B}_{ind}$ will offset this change

A long, straight wire carries a steady current I. A rectangular conducting loop lies in the same plane as the wire, with two sides parallel to the wire and two sides perpendicular. Suppose the loop is pushed toward the wire as shown. Given the direction of I, the induced current in the loop is



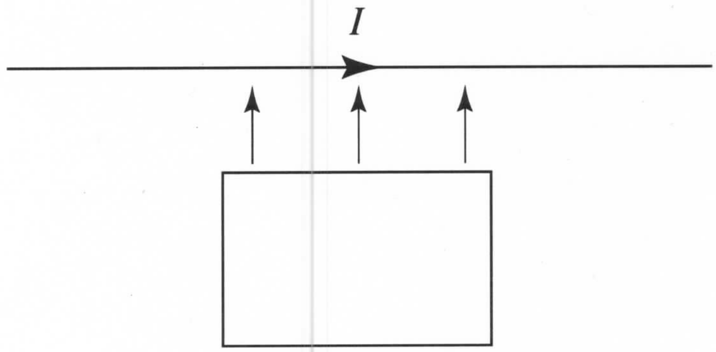
1. Find direction of original $\vec{B}_{wire}$
2. Φ_B increasing?
3. Increasing → $\vec{B}_{ind}$ must point opp way → $\vec{B}_{ind}$ ⊙

Φ_B increasing — moving loop to stronger field

1. clockwise.
2. counterclockwise.
3. need more information

(skipped: just stated repulsive force, talked through directions of forces on loop)

A long, straight wire carries a steady current I. A rectangular conducting loop lies in the same plane as the wire, with two sides parallel to the wire and two sides perpendicular. Suppose the loop is pushed toward the wire as shown. The force on the induced current in the loop by the magnetic field of the wire is



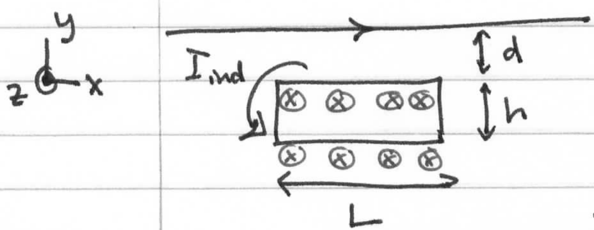
1. repulsive.
2. attractive.
3. need more information

Lenz's law is really about energy conservation: the induced current must flow so that energy is lost, not gained

To illustrate:

$\boxed{\text{CT}}$ force on loop: repulsive

$$\vec{F}_{net} = \vec{F}_{top} + \vec{F}_{bottom} = I_{ind}\,\vec{L}_{top} \times \vec{B}_{\substack{wire\\ at\ top}} + I_{ind}\,\vec{L}_{bottom} \times \vec{B}_{\substack{wire\\ at\ bottom}}$$



$$\vec{B}_{wire} = \frac{\mu_0 I_{wire}}{2\pi r}(-\hat{k})$$

$$\vec{B}_{\substack{wire\\ at\ top}} = \frac{\mu_0 I_{wire}}{2\pi d} \quad \text{and} \quad \vec{B}_{\substack{wire\\ at\ bottom}} = \frac{\mu_0 I_{wire}}{2\pi(d+h)}$$

Top: $I_{ind}\vec{L}_{top} \times \vec{B}_{wire} = I_{ind} L \dfrac{\mu_0 I_{wire}}{2\pi d}(-\hat{j})$ down

Bottom: $I_{ind}\vec{L}_{bottom} \times \vec{B}_{wire} = I_{ind} L \dfrac{\mu_0 I_{wire}}{2\pi(d+h)}\hat{j}$ (up)

Add together $\rightarrow$ net force

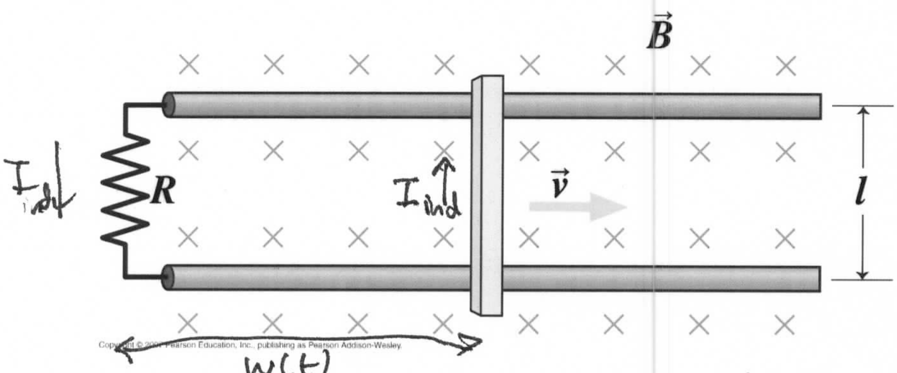
Force <u>must</u> be repulsive or else you could give the loop a nudge and it would start speeding up on it's own!! energy from nowhere.... Instead have to <u>push</u> loop in.

So <u>if</u> induced current is caused by motion (moving loop or moving source of magnetic field), force on I_{ind} <u>opposes</u> that original motion

A pair of parallel conducting rails a distance l apart are connected by a resistor R at the left. A conducting bar can slide across the rails with negligible friction while keeping electrical contact. The whole system is in a uniform magnetic field B as shown. You pull the bar at constant velocity v to the right. (a) What is the direction of the current in the resistor? (b) At what rate do you do work in pulling the bar, and where does the energy go?

(a) 1. Down

2. Up



Work done pulling bar: find force you exert = magnetic force

$$\vec{F} = I_{ind}\vec{L}\times\vec{B}_{external} \Rightarrow F = I_{ind}\, l\, B_{ext}$$

Need I_{ind}: $$|\mathcal{E}_{ind}| = \left|-\frac{d\Phi_B}{dt}\right| = \frac{d}{dt}\left(\vec{B}\cdot\vec{A}\right) = \frac{d}{dt}(BA)$$

$$|\mathcal{E}_{ind}| = B\frac{dA}{dt}$$

$$A = l\,w(t)$$

$$\frac{dA}{dt} = l\frac{dw(t)}{dt} = lv \Rightarrow |\mathcal{E}_{ind}| = Blv, \quad I_{ind} = \frac{|\mathcal{E}_{ind}|}{R}$$

$$F = \left(\frac{Blv}{R}\right)lB = \frac{B^2l^2v}{R}$$

Work you do $= \vec{F}\cdot\Delta\vec{r} = F\Delta r$

Rate of work $= \frac{d}{dt}(F\Delta r) = F\frac{d}{dt}(\Delta r) = Fv = \frac{B^2l^2v^2}{R}$

11:50 to here

Problem (Wolfson Ch 27 #47):

(a) Ask: direction of current?

Now flux is increasing by area of loop actually increasing:

$$\Phi_B = \vec{B} \circ \vec{A}(t) = BA(t)$$

so $$\frac{d\Phi_B}{dt} = \frac{d}{dt}(BA(t)) = B\frac{dA(t)}{dt}$$

increasing flux into page → current flows to produce $\vec{B}_{ind}$ in opposite direction → current is CCW

(b) Work done pulling bar: force you exert must equal mag force $\vec{F} = I_{ind}\vec{L} \times \vec{B}$ in opp direction

induced current:

$$I_{ind} = \frac{|\mathcal{E}_{ind}|}{R} = \frac{1}{R}\left|\frac{d\Phi_B}{dt}\right| = \frac{1}{R}B\frac{dA(t)}{dt}$$

again $A(t) = l\,w(t)$

$$\frac{dA(t)}{dt} = l\frac{dw(t)}{dt} = lv$$

$$\Rightarrow I_{ind} = \frac{Blv}{R}$$

⇒ magnitude of force is $F = I_{ind}LB$ b/c $\vec{B} \perp$ current

$$F = \left(\frac{Blv}{R}\right)lB = B^2l^2v/R$$ constant force!

length of bar feeling force

Work is (done by you) $\int \vec{F} \circ d\vec{r} = \vec{F} \circ \Delta\vec{r}$ for const force $= F\Delta r$ b/c force in same dir as motion

and $\Delta\vec{r} = vt$ how fast you pull bar · time pulling

$$\rightarrow W_{by\ you} = \left(\frac{B^2l^2v}{R}\right)(vt)$$

Rate of work $= \frac{dW}{dt} = \frac{B^2l^2v^2}{R}$ = power

Where does the energy go? into resistor:

$$I^2R = \frac{B^2\ell^2v^2}{R^2}R = \frac{B^2\ell^2v^2}{R}$$

So you pull bar
→ magnetic force on charges → $\vec{E}$ in bar due to charge separation → drives current around loop
→ dissipates energy as heat

All the examples we're done so far are for a moving conductor in $\vec{B}$ — understand source of current as magnetic force

Physics says that laws of physics can't depend on whether one object is moving at constant speed & other is stationary

→ must get same result if magnetic field is moving and conductor is stationary!

~~then the surprise is~~

Turns out: a changing $\vec{B}$ at some location in space causes an "induced $\vec{E}$" to exist — we'll discuss in depth on Tuesday (when no exam hanging over your head)

So not only do we get current from moving magnets — also from time-dependent $\vec{B}$:

$\mathcal{E} = -\frac{d\Phi_B}{dt}$ applies ~~[illegible]~~ however we get $\frac{d\Phi_B}{dt}$

- moving conductor
- moving magnetic field source
- time-dependent currents producing magnetic field ~~source~~

These are the major problem with pacemakers in MRI machines
- person moves in the MRI - moving circuit → induced current that interferes w/pacemaker function
- MRI fields are pulsed (time varying) ~~[illegible]~~ w/very rapid changes

$\boxed{12{:}10}$

$\boxed{\text{Loop CT}}$:

~~[illegible]~~ Decreasing current in bottom loop → decreasing upward flux through top loop

⇒ top loop's I_{ind} must flow to produce upward $\vec{B}_{ind}$ inside top loop

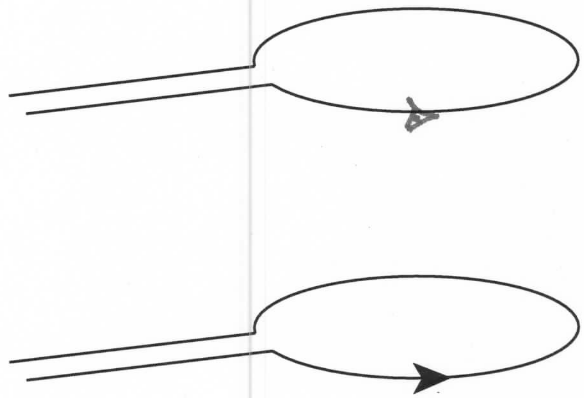
⇒ current must flow same way

if instead ~~top~~ bottom loop has increasing current: top loop has opposite I_{ind}

Useful: basis for wireless charging — oscillating current ~~[illegible]~~ → oscillating $\vec{B}$ → induces current in the charging circuit

We'll start off Tuesday w/a problem on this.
(Farmer problem)

A conducting ring is held a certain distance above a loop carrying a *decreasing* current as illustrated below. As viewed from above, the current through the bottom loop induces an emf in the top ring that causes a current in the following direction

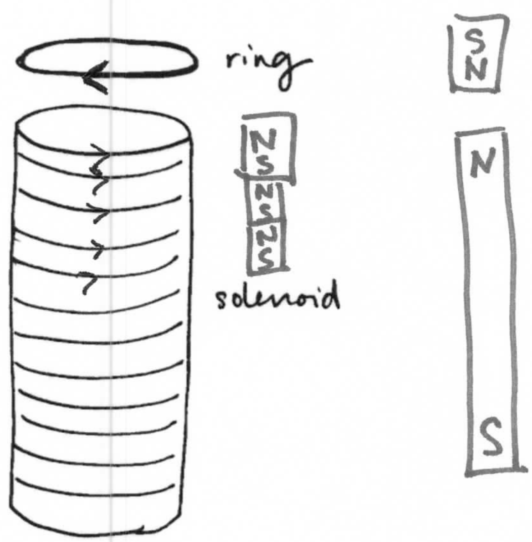


I decreasing with time
(CCW) ⇒ flux decreases
→ upper loop's $\vec{B}_{ind}$ in same direction

1. clockwise
2. counterclockwise
3. the answer depends on the distance between the two
4. none of the above

→ upper loop's I in same direction

A conducting ring is set on top of a solenoid that initially carries no current (so the solenoid has no magnetic field). Then, the current to the solenoid is turned on.



$I_{ind} = \frac{\mathcal{E}_{ind}}{R}$

When current begins to flow in the solenoid, the ring feels a:

1. force toward the solenoid
2. force away from the solenoid
3. depends on which way the solenoid current flows.

Jumping ring demo: example of time-varying field

Warning: motion is not causing the change in flux when ~~fields change due to~~ source currents ~~change~~ producing fields are changing

{ So must think about $\frac{d\Phi_B}{dt}$ not about forces opposing motion }

$\boxed{CT}$ force on ring: repulsive! (otherwise no demo :))

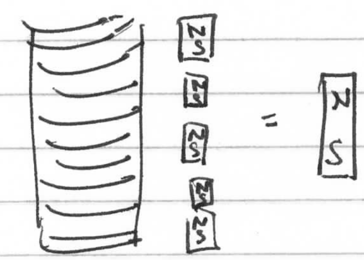
induced current:

solenoid current increases $\rightarrow \vec{B}$ increases

induced current therefore must produce opposite $\vec{B}_{ind}$

$\rightarrow \vec{\mu}_{ind}$ is opposite solenoid $\vec{B}$

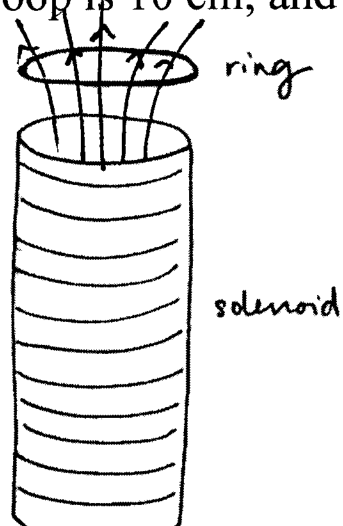
Think of solenoid as long stack of bar magnets:



ring's $\vec{\mu}$ is opposite $\Rightarrow$ repulsive!

Problem: you've done/will do very similar problem in lab to give us more time today, will post detailed solution, let me know if you have questions

For the "jumping ring" demonstration, find the induced current in the ring if the magnetic field strength changes at 120 T/s, the maximum magnetic field strength is 1.0 T, the diameter of the loop is 10 cm, and the resistance of the ring is 0.01 Ω.



To find induced current we use

$$I_{ind} = \frac{\mathcal{E}_{ind}}{R} = \frac{1}{R}\left|\frac{d\Phi_B}{dt}\right|$$

To find Φ_B through the ring:

$\vec{B}$ of the solenoid is sketched

Through the ring, $\vec{B}$ is nearly vertical

so approximate $\vec{B}$ as uniform and vertical through loop

$\Rightarrow \Phi_B = \int \vec{B} \cdot d\vec{A}$ is approximately $\vec{B} \cdot \vec{A}_{Loop} = BA_{loop}$

because $\vec{A}_{Loop}$ is also vertical

(so $\vec{B} \cdot \vec{A}_{Loop} = BA_{Loop} \cos 0° = BA_{Loop}$)

$$\frac{d\Phi_B}{dt} = \frac{d}{dt}(BA_{Loop}) = \frac{dB}{dt} A_{Loop}$$

We are given $\frac{dB}{dt} = 120\ T/s$ and $A_{Loop} = \pi r_{loop}^2 = \pi (0.05\,m)^2$

$R_{Loop} = 0.01\Omega$

Substituting values

$$\Rightarrow I_{ind} = \frac{1}{R}\frac{dB}{dt}A_{Loop} = 94\ A!!$$