**Announcements 4/27/10**

Problem set 12 is due Thursday in class; SA session tonight in SC L32 (note change of room)

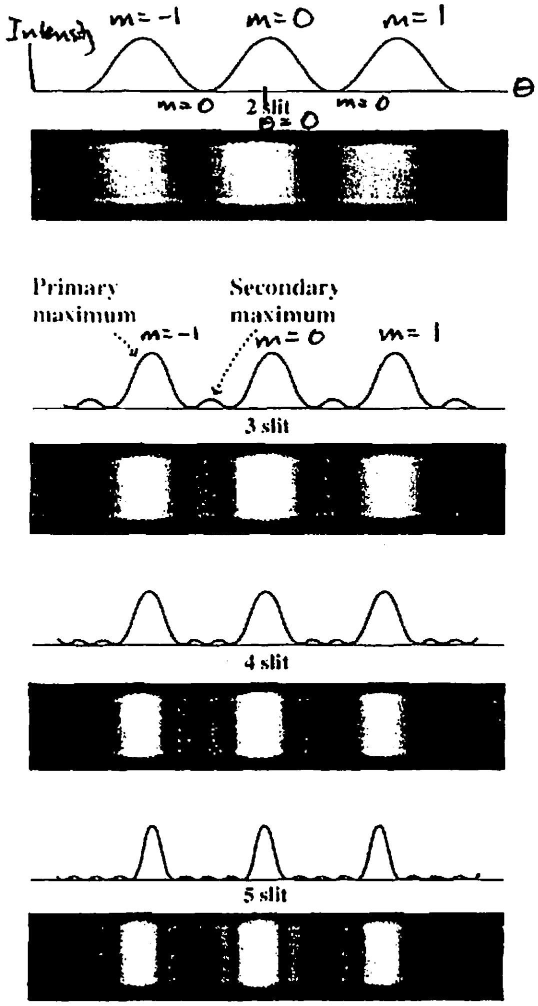
On PS 12, Ch. 32 Problem 45 is changed to extra credit.

PS 13 will be posted later today, due Wednesday next week.

Reading for today: 32.6, skip calculation of intensity in single-slit diffraction
No additional reading for Thursday

Online reading question today: list a few different topics we could discuss Thursday, let me know what interests you, optional

Intensity patterns






## Multiple slit interference

Slit width a smaller than $\lambda$

BUT slit separation d larger than $\lambda$

Pattern depends only on d, not a

All primary maxima of same brightness

$N=2$:

maxima at $d \sin\theta = m\lambda$

minima at $d \sin\theta = (m \pm \frac{1}{2})\lambda$

+ for positive m
- for negative m

$N>2$:

primary maxima still at $d \sin\theta = m\lambda$

large N → brighter because combining light from many slits, AND ~~brightness~~ light spread over smaller area

## Single slit diffraction:

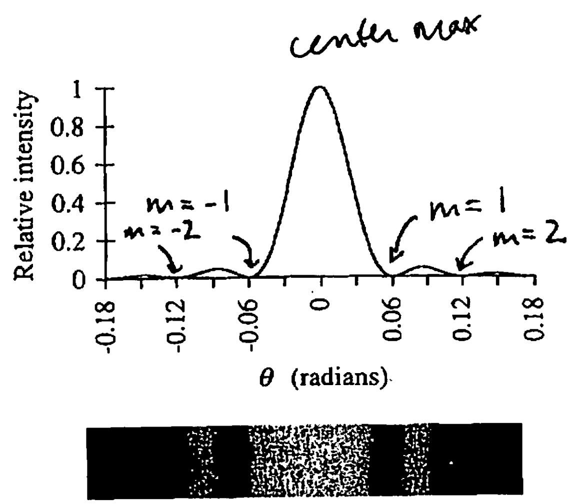
Slit width a greater than $\lambda$

Central bright maximum, otherwise maxima difficult to locate, decrease as go out from center

Minima correspond to

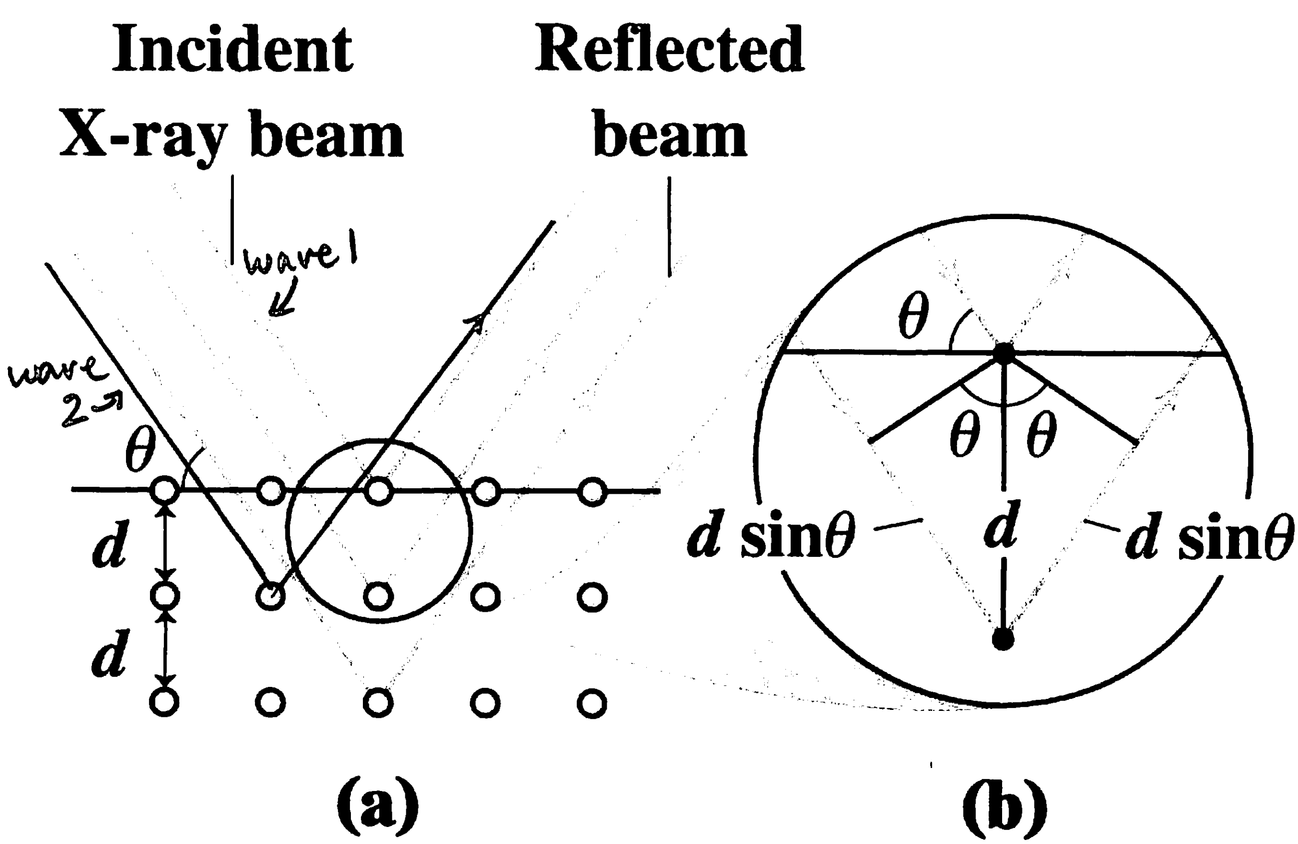
$$a \sin\theta_m = m\lambda \quad \text{for } m \neq 0$$

(integer m)




Pattern on screen due to *single* slit

Wrapup of Xray diffraction: marked waves 1 & 2 are what interfere






$\Delta r = 2d\sin\theta$

4/27/10

Summary from last time:

many cases of interference between multiple point sources
- double slit
- multiple slit
- X-ray diffraction to measure spacing between crystal planes
- antireflection coatings

In all cases: obtain constructive interference when phase differences are an integer number of cycles

$$\Delta r = m\lambda \Rightarrow \phi = k\Delta r = 2m\pi$$

destructive interference when phase differences are a half-integer number of cycles

$$\Delta r = (m+\tfrac{1}{2})\lambda \Rightarrow \phi = k\Delta r = 2\pi(m+\tfrac{1}{2})$$

Assumes all sources start out in phase
all have same polarization
all spread spherically

In practice: achieve by shining plane wave on small slits (slitwidth $a \leq \lambda$); light from each slit = point source

J.J.

4/27/10

Today

assume slit ~~size~~ width $a \lesssim \lambda$

(1) Wrap X-ray diffraction

(2) Thin film interference

phase change at reflection:

$\phi_{hard} = 180°$ (inverts) if $n_1 < n_2$ fast → slow

$\phi_{soft} = 0°$ if $n_1 > n_2$ slow → fast

$\Rightarrow \phi = k\Delta r + \Delta\phi_{refl}$ $\boxed{\text{phase difference between waves}}$ $\Delta\phi_{refl} = \phi_{refl\,2} - \phi_{ref}$

destructive interference → min reflection

~~max refl~~ constructive interference → max reflection

Effect of slit size

(3) Diffraction through a single opening

What if slit width $a$ is not $\lesssim \lambda$ ?

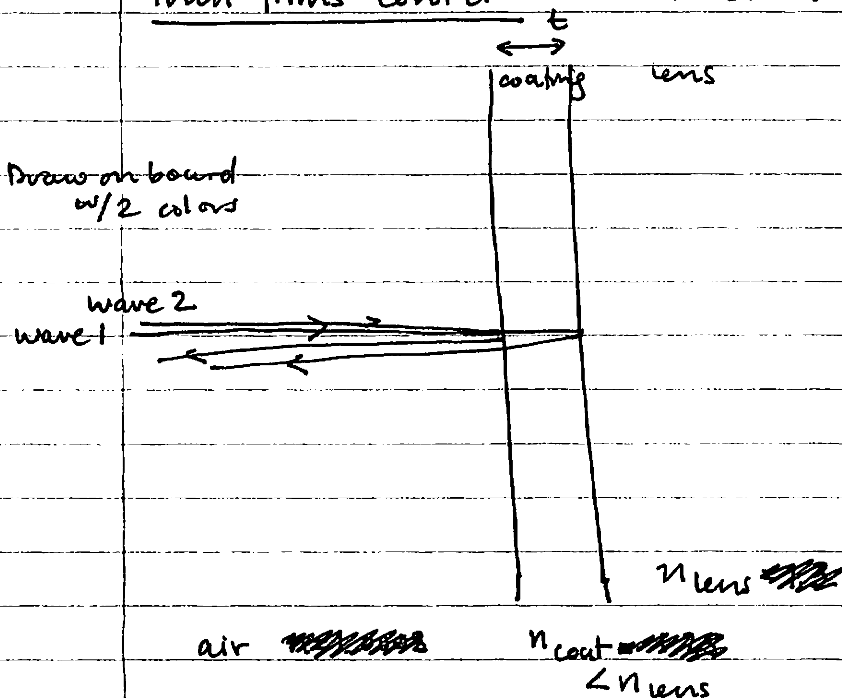
Observe pronounced brightness variations when light passes through aperture with $a \simeq 2\text{-}100\lambda$

- pattern depends on $a$

Maximum ~~at~~ center ($\theta = 0°$)

Minima at $a \sin\theta_m = m\lambda$ for $m \neq 0$ (integer)

Thin films cont'd: antireflection coating




NOTE: book gives a single formula for "thin films" — doesn't apply if film is between two different materials! Just think about phase diff

To get minimum reflection want destructive interf. between wave 1 & wave 2

$\boxed{CT}$ thickness of coating?

minimize reflection: want $\phi = 180°$ (out of phase)

both reflections are "hard" so both pick up 180° shift

→ cancels (no difference)

[ explicitly if asked:

$$E_1 = E_p \sin(kr_1 - \omega t + \phi_{refl\,1})$$

$$E_2 = E_p \sin(kr_1 - \omega t + k\Delta r + \phi_{refl\,2})$$

$$\phi = \text{phase}_2 - \text{phase}\,1 = k\Delta r + \phi_{refl\,2} - \phi_{refl\,1}$$

if $\phi_{refl\,2} = \phi_{refl\,1} = 180°$ then cancel ]

Therefore phase difference just comes from $\phi = k\Delta r$

So thickness just corresponds to path difference $\Delta r = \lambda/2$

BUT must be $\lambda_{coat}$

$$\Rightarrow 2t = \lambda_{coat}/2 \Rightarrow t = \frac{\lambda_{coat}}{4}$$

$\Delta r = 2t$

Problem: we have $n_{air} < n_{coat}$ and $n_{coat} < n_{lens}$ so above applies

Coating thickness:

$$t = \frac{\lambda_{coat}}{4} = \frac{\lambda_{vacuum}}{n_{coat}4} = \frac{500 \times 10^{-9}\,m}{1.25 \cdot 4} = 100 \times 10^{-9}\,m = 100\,nm \;(!!)$$

To make a coating to minimize reflection of light of a particular wavelength, the thickness of the coating should be:

1. $\lambda_{vacuum}$
2. $\lambda_{vacuum}/2$
3. $\lambda_{vacuum}/4$
4. $\lambda_{coating}$
5. $\lambda_{coating}/2$
6. $\lambda_{coating}/4$

recall $\lambda_{coating}$ = wavelength in coating = $\dfrac{\lambda_{vacuum}}{n_{coating}}$

We have two "hard" reflections because $n_{coat} < n_{lens}$ and $n_{air} < n_{coat}$

$\rightarrow \quad \phi_{refl\,2} = \phi_{refl\,1} = 180°$

$\rightarrow \quad \phi = k\Delta r + \underbrace{\phi_{refl\,2} - \phi_{refl\,1}}_{\text{cancel}} = k\Delta r$

and so we want destructive interference just due to path length

$\rightarrow \quad \Delta r = \dfrac{\lambda_{coat}}{2}$ needs to be $\lambda_{coat}$ b/c extra distance traveled by wave 2 is in coating

$\Delta r = 2t \Rightarrow \quad 2t = \dfrac{\lambda_{coat}}{2} \Rightarrow \quad t = \dfrac{\lambda_{coat}}{4}$

You are ordering new glasses and want to minimize reflection of light from the glasses. The material used for the lenses has index of refraction $n_{lenses} = 1.3$; you arrange to have the glasses coated with a transparent material with index $n_{coating} = 1.25$. How thick should the coating be to minimize reflection of 500 nm light?

Check: $n_{lens} > n_{coat}$ ☑

$$t = \frac{\lambda_{coating}}{4} = \frac{\lambda_{vacuum}}{n_{coat}\,4} = \frac{500 \times 10^{-9}\,m}{1.25 \cdot 4} = 100 \times 10^{-9}\,nm$$

If $n_{coat} > n_{lens}$?

→ this gives us constructive interference

$$\phi = k\Delta r + \underbrace{\cancel{\phi_{refl\,2}}}_{0^\circ} - \underbrace{\cancel{\phi_{refl\,1}}}_{180^\circ}$$

need $\Delta r$ to be $\lambda$ to compensate for the difference

What if $n_{coat} > n_{lens}$?

Then $\phi_{refl\,2} = 0°$

$\phi_{refl\,1} = 180°$

and we need path length difference to be a full wavelength

$\Rightarrow \quad 2t = \lambda_{coat} \Rightarrow t = \dfrac{\lambda_{coat}}{2}$ twice as thick

Choose $n_{coat} < n_{lens}$ to get thinner coatings.

Butterfly iridescence

Max reflection comes from constructive interference

Constr intf of certain colors $\Rightarrow$ brilliant iridescence in Morpho butterfly, colors on soap bubbles / oil films

Show YouTube video

Get constructive interference of blue light $\lambda = 440\text{ nm} = 0.440\,\mu m$

Why? Handout: wings made up of scales w/ $\lambda$-scale structure like X-ray diffraction

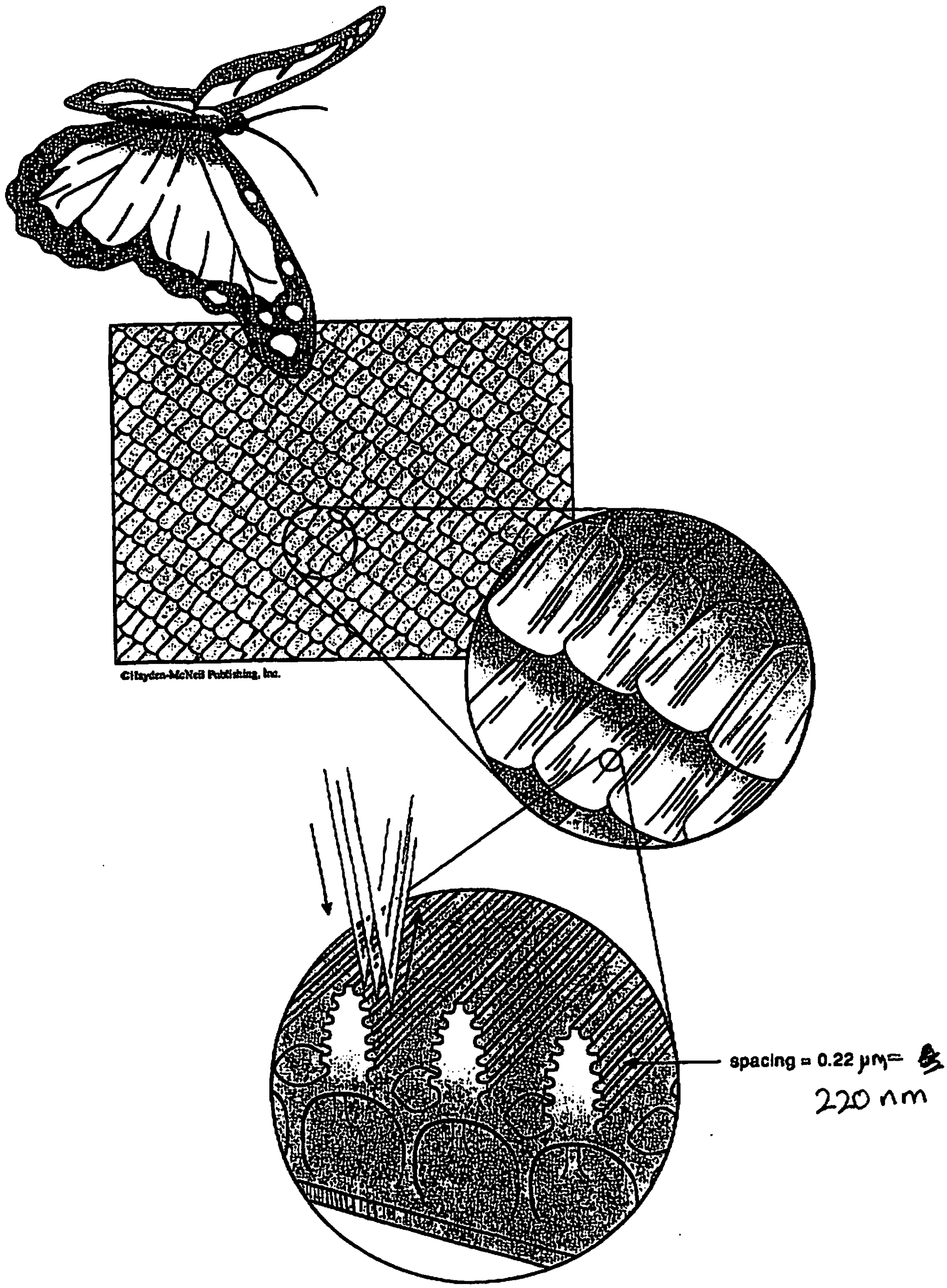
- light @ certain angles reflects from adjacent ridges and interferes constructively: both are hard refl $\Rightarrow \Delta\phi_{refl} = 0$
- $d = 220$ nm

so constr interference when $\Delta r = m\lambda$

close to normal incidence, $\Delta r \approx 2d \Rightarrow$ constr int. of blue light!

skipped $\boxed{\text{CT}}$ Glass slide covering one opening $\rightarrow \phi = 180°$

switches locations of bright & dark


©Hayden-McNeil Publishing, Inc.
spacing = 0.22 μm =
220 nm

## Diffraction through a single slit

At the beginning of this section we assumed the slit width $a$ was comparable to $\lambda$ and it turned out not to affect anything

In practice with slits that are similar to but larger than $\lambda$: $a$ does matter

Specifically: if $a \approx 2\lambda - 100\lambda$ we get interference of different parts of the wave passing through the slit

Consequences are enormously important for surprising reasons: turns out to limit how small a focused spot of light can be!

Let's see why — to understand must begin w/ an idea called Huygens' Principle after its discoverer

Any wave consists of a set of point sources ("wavelets") very close together

on each wave crest, all wavelets are at their crests

as wave travels, each wavelet spreads in all directions

GO SLOWLY CONTRAST (a) + (b) in top panel

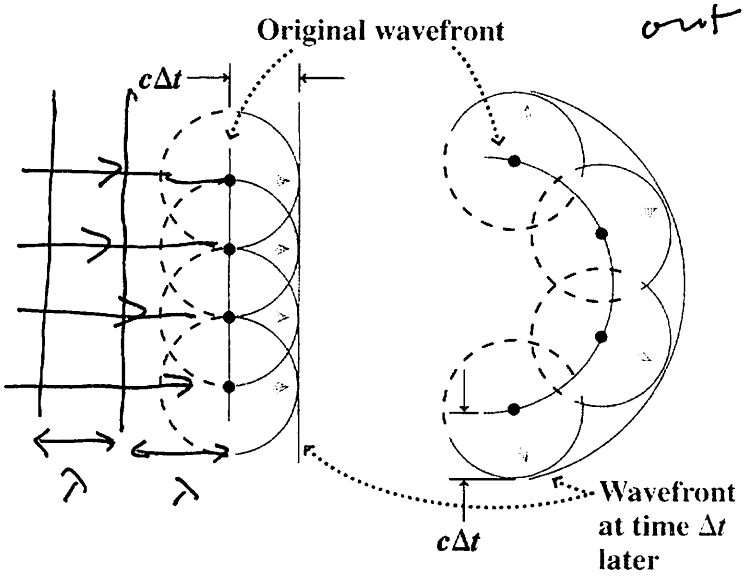
Show Wolfson figure: line of wavelets stays a long line except at edges

BUT get spreading at edges

If $a \sim 2-100\lambda$ the spreading is noticeable

DEMO: green laser through single slit

Plane wave = many very closely spaced point sources, each of which spreads out in all directions

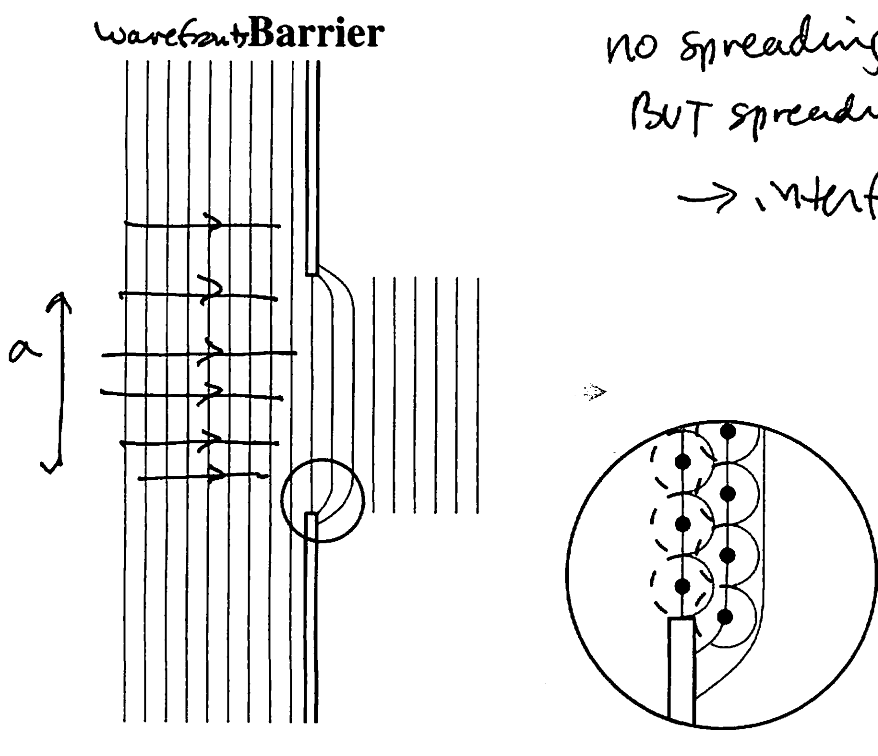



**(a)** **(b)**



If we have a long line of these point sources (a) $\Rightarrow$ no spreading b/c net effect is for all to propagate forward

But a curved set of point sources (b) now allows spherical spreading




**(a)**



no spreading in middle
BUT spreading @ edges
$\rightarrow$ interference if $a \sim 2\text{-}100\,\lambda$

## What produces this pattern?

Think of wave passing through slit as set of closely spaced point sources

At $\theta = 0°$ (straight ahead)
each source is in phase with the one on the other side of the center line $\Rightarrow$ maximum

Rest of maxima are hard to find – can't use approach we took w/ many slits b/c we don't know sep btw sources (not well defined)

Can find minima:

* Divide slit in half

Top source in upper half will be $180°$ out of phase w/ top source in bottom half when path length diff is $\lambda/2$

Distance between them is $a/2$

$\Rightarrow \frac{a}{2}\sin\theta = \frac{\lambda}{2}$ gives destructive: ~~m = ±1 minimum~~

like d for 2-source $\Rightarrow a\sin\theta_1 = \lambda$ $1^{st}$ min

Divide slit in four: destructive if

$$\frac{a}{4}\sin\theta_2 = \frac{\lambda}{2} \Rightarrow a\sin\theta_2 = 2\lambda \qquad 2^{nd} \text{ min}$$

Divide into any even # of parts

$$\Rightarrow \frac{a}{2m}\sin\theta_m = \frac{\lambda}{2} \Rightarrow a\sin\theta_m = m\lambda$$

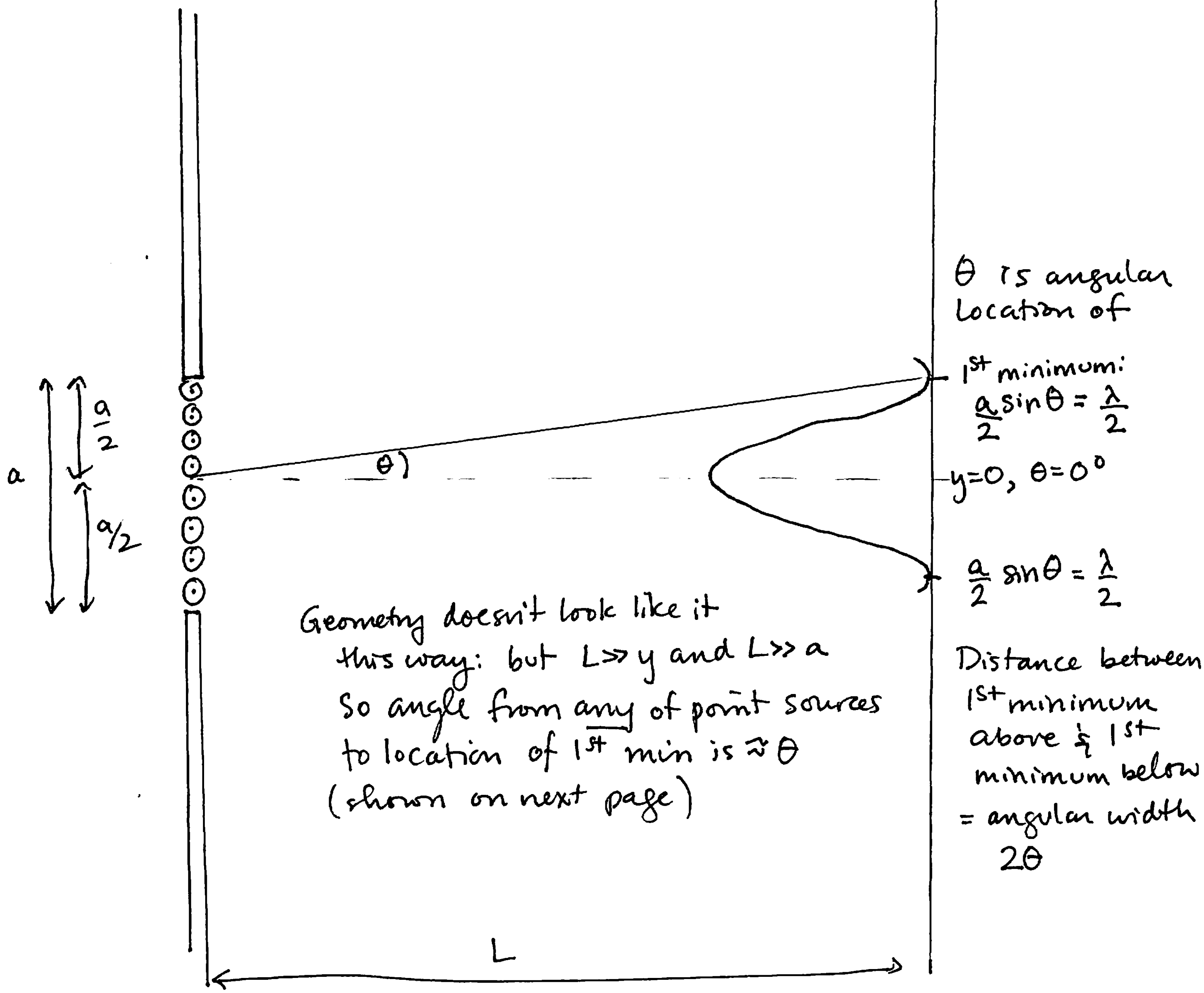
$m^{th}$ minimum

Looks like double/multiple slit maximum!

BUT: assumes $a \gtrsim \lambda$
depends on slit width
identifies location of dark spots ("fringes")

Show handout

Slit of width a illuminated by plane wave: slit filled with point sources

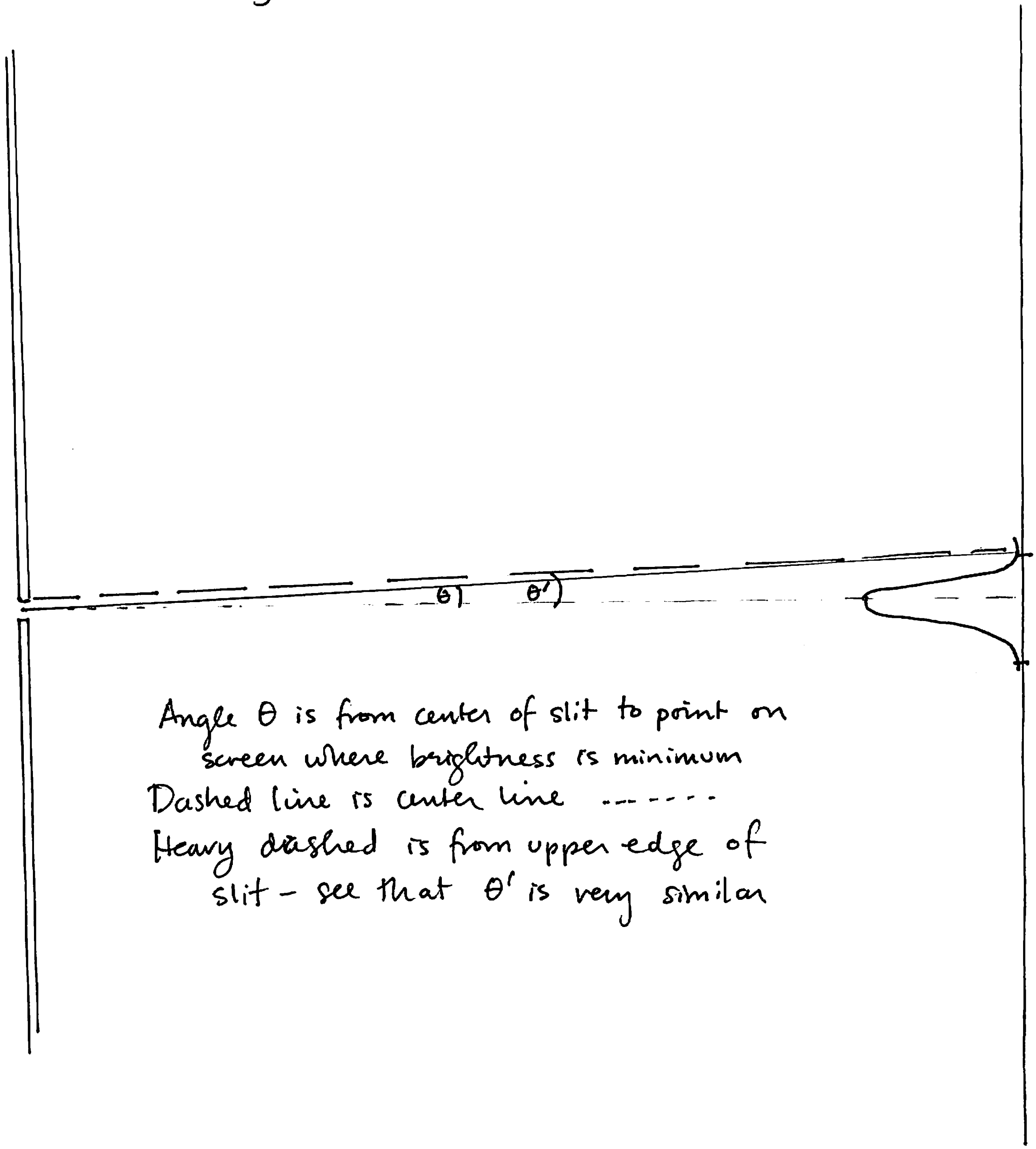



At center of screen ($y=0,\ \theta=0$): max brightness
pairs of sources above/below center line are in phase
→ add constructively

Other maxima very difficult to find – don't know d (not well defined)

Instead find minima:
think of dividing slit in half vertically
destructive if top source in top half interferes destr. w/top source in bottom half, and so on for all pairs

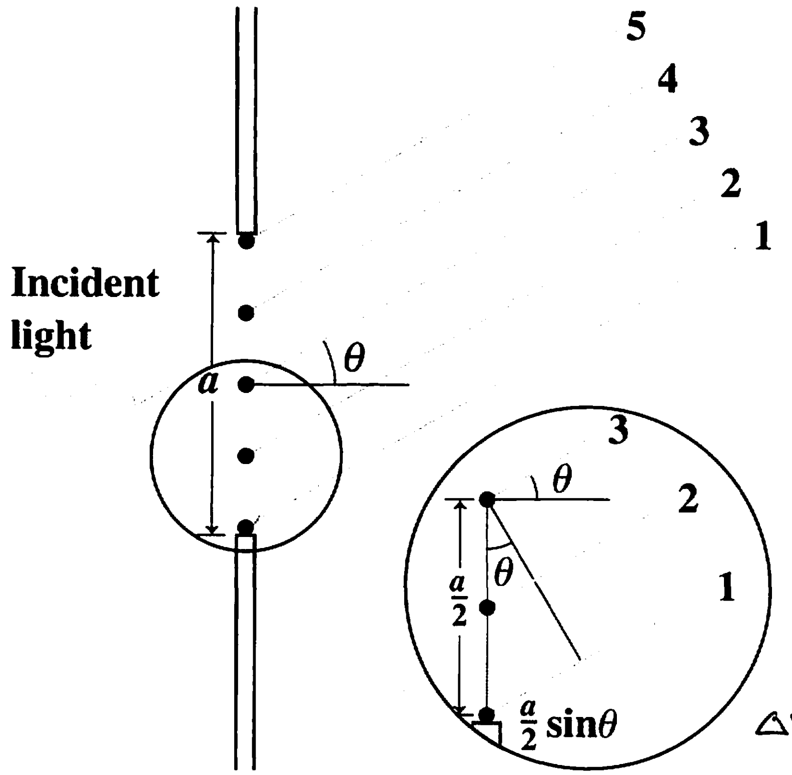
What this really looks like:




Angle $\theta$ is from center of slit to point on screen where brightness is minimum

Dashed line is center line - - - - - -

Heavy dashed is from upper edge of slit - see that $\theta'$ is very similar




**(a)** **(b)**



$\Delta r = \frac{a}{2}\sin\theta$

like two slits separated by d

destructive intf:

$\Delta r = \frac{a}{2}\sin\theta = (m+\frac{1}{2})\lambda$

Choose smallest $\Delta r$ $(m=0)$

$1^{st}$ min at $\Rightarrow \frac{a}{2}\sin\theta = \frac{\lambda}{2}$

$\Rightarrow a\sin\theta = \lambda$

DESTRUCTIVE INTERF. : minimum

Subdivide slit in 4

$\Delta r = \frac{a}{4}\sin\theta = \frac{\lambda}{2} \Rightarrow a\sin\theta = 2\lambda$

.... $a\sin\theta_m = m\lambda$ destructive EXCEPT NOT $m=0$

Light falls on a single narrow slit and forms a diffraction pattern on a screen. If the slit is replaced with a slit of half the width, the central maximum in the resulting pattern is

1. half the angular width of
2. twice the angular width of
3. brighter but the same shape as
4. dimmer but the same shape as

the original pattern.

narrower slit → wider pattern

1st min.

$a \sin\theta = \lambda$

$\sin\theta = \dfrac{\lambda}{a}$

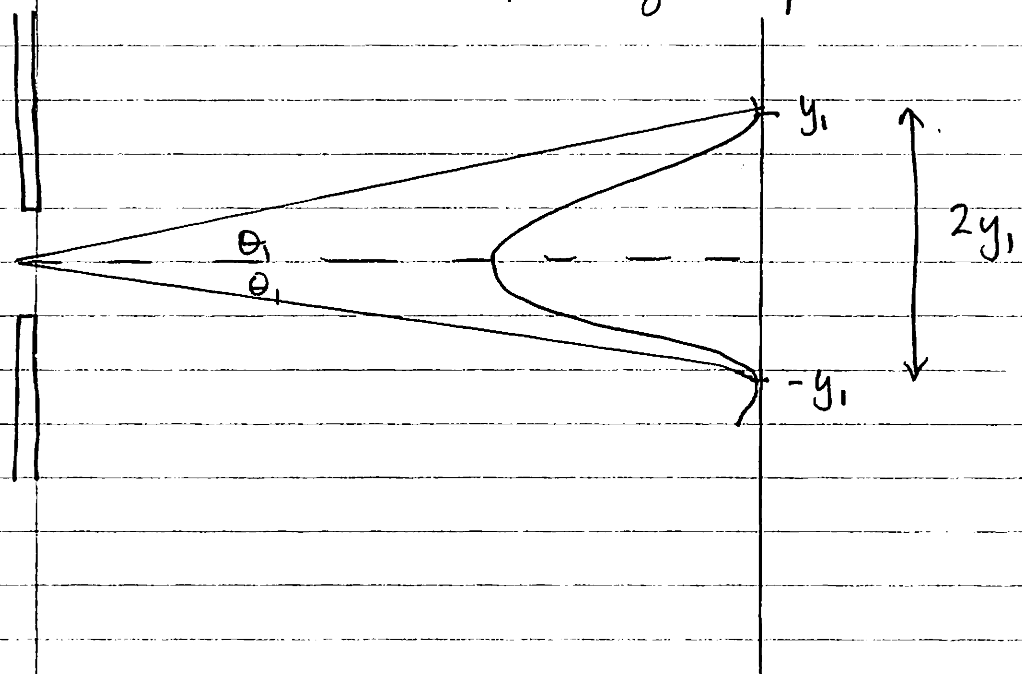
angle @ 1st min get bigger as a gets smaller

How does this pattern depend on slit width?

$\boxed{CT}$ Smaller slit → wider spread

angular width of central max $= 2\theta_1$

with $\theta_1$ = angular position of $1^{st}$ min

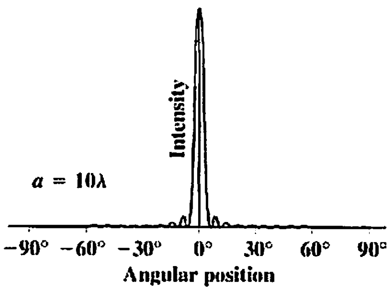



How does $y_1$ relate to $\theta_1$? $\sin\theta_1 = \dfrac{y_1}{\sqrt{y_1^2+L^2}} \approx \dfrac{y_1}{L}$ for $L \gg y_1$

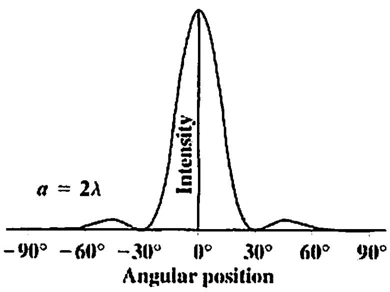
We know $a\sin\theta_1 = \lambda$

so $a\dfrac{y_1}{L} \approx \lambda \Rightarrow y_1 \approx \dfrac{\lambda}{a} L$

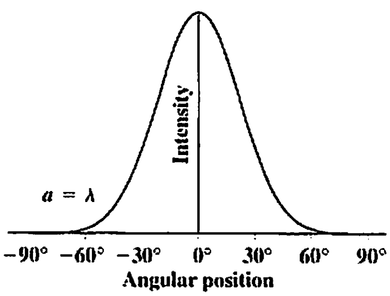
smaller $a$ → bigger $y_1$


Intensity
a = 10λ
−90° −60° −30° 0° 30° 60° 90°
Angular position

(a)


Intensity
a = 2λ
−90° −60° −30° 0° 30° 60° 90°
Angular position

(b)


Intensity
a = λ
−90° −60° −30° 0° 30° 60° 90°
Angular position

(c)

Green laser light ($\lambda = 532$ nm) is incident on single slit of width 0.16 mm. In terms of the distance from the slit to the screen, how far apart are the two first-order minima in the diffraction pattern?

Use result $y_1 \approx \frac{\lambda}{a} L$

Note distance between minima is $2y_1$

$$\rightarrow \quad 2y_1 = \frac{2\lambda}{a} L = \frac{2(532 \times 10^{-9} \text{m})}{(0.16 \times 10^{-3} \text{m})} L$$

$$= 6.7 \times 10^{-3} L$$