Announcements 2/9/10 (always posted with class notes)

As announced last Thurs, additional problem 2 from PS 3 is due Thursday with the self-test. However, if you finished additional problem 2 already, you can turn it in with your problem set.

Ranking forces problem from last Thurs: covered in lab

PS4 will be due Thursday 2/18, posted online later today, on the last material before midterm Monday 2/22.

Please don't hesitate to come to my office hours, come to my Thursday night problem session or the SA sessions, or make an appointment to ask questions.

Reading for Thursday:
section 21.6 p. 359- end of second line on p. 360, AND **22.1**

We are not going to cover Gauss's Law (most of Ch. 21). So, you are not responsible for anything concerned with Gauss's Law.

Next Tuesday: **22.2**, omitting "Continous charge distributions" and Ex. 22.6 and 22.7, and **22.3** only up through "Got it?" 22.6

Wolfson passes over potential energy blindingly fast in 22.1; I am going to spend more time making the connection to potential energy.

Key ideas from last time

To describe the effect of lots of charges on the space around them, we introduce the electric field

Any arrangement of charge ("source charges") fills the space around it with an electric field.

The field at any location in space tells us the force per charge that a different charge (the "test" charge, or the charge feeling the force) would feel if placed at that point:

if $\vec{E}$ is the field of the sources, q is the charge of the test charge, and $\vec{r}$ is the location where we want to know the field or force:

$$\vec{E}(\vec{r}) = \vec{F}_{on\,q}/q \qquad \text{and} \qquad \vec{F}_{on\,q} = q\vec{E}(\vec{r})$$

Key: once we've found $\vec{E}$ once for a particular arrangement of charge, we can just use it again; don't need to keep recalculating!

Van de Graaff generator

Figure 30.4 shows a schematic diagram of a Van de Graaff generator — a mechanical device invented in the 1930's by Robert J. Van de Graaff to create large electrostatic charges. The basic principle is extremely simple: a nonconducting belt delivers charge carriers to a hollow conducting dome that rests on a nonconducting support.

There are three important steps in the operation of the generator. The first step involves a transfer of charge carriers to the belt at *A* — this transfer can be done by literally "spraying" charged particles onto the belt or simply by rubbing the rubber belt against some appropriate material.

The second step involves a transport of the charge carriers to the dome. This step is possible because the belt is nonconducting and so the charge carriers are not mobile — they are stuck to the belt, which is driven by a motor around a pulley inside the dome. The motor must do work on the charge carriers to move them against the electrostatic field of the dome. (In the example shown in the diagram, positive charge carriers at *B* must be transported upward against the downward electric field of the positively charged dome.)

The third step involves transferring the charge carriers from the belt onto the dome. As we saw in Section 28.5, the electric field inside a hollow conductor is always zero, and any charge inside a conductor moves toward the outer surface. So, once the charge carriers are inside the dome, they tend to move to the outer surface of the dome. For this purpose a comb of conducting needles is placed close to the belt at *C*. If the charge carriers on the belt are electrons, the electrons hop onto the brush and move via the connecting wire toward the outside of the metal dome causing the dome to acquire a negative charge. Alternatively, the charge carriers on the belt can be positively ionized air molecules, in which case electrons in the comb are attracted toward the ions. These electrons then jump from the comb onto the belt, neutralizing the ions on the belt while leaving a positive charge behind on the outside of the dome.

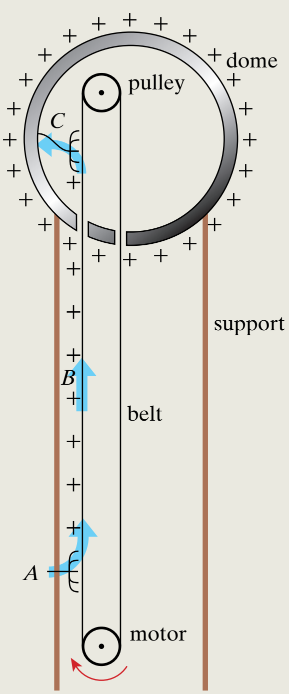


Figure 30.4 Schematic diagram of a Van de Graaff generator.

Construction of a huge double Van de Graaff generator for the MIT physics department in New Bedford, Massachusetts in 1935. These generators, currently at the Boston Museum of Science, generated opposite charges and were able to produce potential differences of 10,000,000 volts between the two 4.5-m domes.

J.J.

2/9/10

Today's goals:
continue to
(1) Find fields for four benchmark arrangements ("distributions")
of charge: dipole, point/sphere, line, plane

(2) In the process: introduce field line representation of $\vec{E}$

(3) Polarization

Discuss along w/ summary sheet

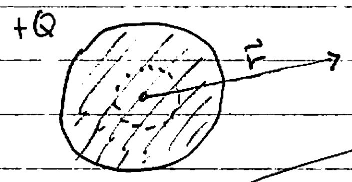
Purpose of $\vec{E}$:
if we have a particular arrangement of charge, such as a sphere, and calculate its $\vec{E}$ everywhere, then we can use that result to find
(a) force on any other charge in its vicinity
(b) potential energy and potential (Thurs)

(from summary page: $\vec{E}(\vec{r}) = \vec{F}/q$, $\vec{F}_{on\, q} = q\vec{E}(\vec{r})$ if q is at $\vec{r}$)

Goal is therefore to find $\vec{E}$ for four typical arrangements of charge, use them over & over!

Spherical (from last time): $\vec{E}(\vec{r}) = \dfrac{kQ}{r^2}\hat{r}$ with $\vec{r}$ measured from ctr of sphere
sphere w/ total chg Q arranged uniformly

Same as if all charge at a point @ ctr!



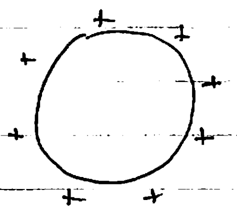
Why is this? How would we prove it?
Calculate same way as we will shortly for line of charge: by adding up dividing up into bits, $\vec{E}$ from each bit

Long, tedious calculation — but key idea is that symmetry of this arrangement means bits all at same distance from center have fields that add up to cancel the differences from being @ ctr.

Example: van de Graaf generator

Belts inside rub on something → get charged (lose or gain e^-)
Belts then rotate around, carry excess charge to metal dome
Charge can move to/from dome to make belt neutral again;
net charge spreads uniformly over dome (will discuss why Thurs/next Tues)

Show demo strings attached between halves work really well! ⇒



treat as spherical shell (hollow sphere) of charge

$$\vec{E}(\vec{r}) = \frac{kQ}{r^2}\hat{r} \quad \text{for } r > \text{sphere radius}$$

$\boxed{\text{CT}}$ field strengths: field is $\underline{\underline{9x}}$ as strong as at P

surface : $r = 0.10$ m

P : $r = 0.30$ m

so $E(r=0.10\text{ m}) = \dfrac{kQ}{(0.10\text{m})^2}$

$E(r=0.30\text{ m}) = \dfrac{kQ}{(0.30\text{m})^2} = \dfrac{E^{(r=0.10)}}{3^2}$ Denom is 9x greater → E is 9x less

How do we use this field information? Can find forces.
Force on e^- in an air molecule just above surface of vdG:
if vdG has lost 1×10^{-12} mol (= 1 pmol) of e^-

$(1\times10^{-12}\text{ mol } e^-)(6.023\times10^{23}/\text{mol}) = 6\times10^{11}\ e^-$

Total charge is thus $(6\times10^{11}\ e^-)(1.609\times10^{-19}\ C/e^-) = 1\times10^{-7}$ C

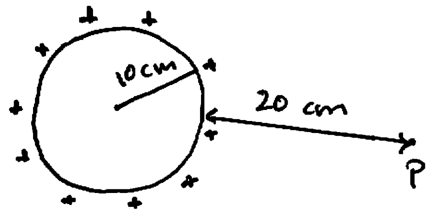
and then at surface, strength of $\vec{E}$ is → $E = \dfrac{kQ}{r^2} = \dfrac{(9\times10^9\text{ N m}^2/\text{C}^2)(1\times10^{-7}\text{ C})}{(0.10\text{ m})^2} = 9\times10^4\ \text{N/C}$

force on an electron is attractive and mag is

$$F = qE = (1.609\times10^{-19}\ \cancel{C})(9\times10^4\ \text{N}/\cancel{C}) = 1\times10^{-14}\text{ N}$$

tiny force but almost enough to ionize air — ionizes at "breakdown field"

Consider the van de Graaff generator to be spherical with radius 10 cm. How does the strength of the field at the surface of the van de Graaff compare to the strength of the field at point P, 20 cm from the surface?



1. At the surface, the field is four times as strong as at P.
2. At the surface, the field is twice as strong as at P.
3. At the surface, the field is the same strength as at P.
4. At the surface, the field is half as strong as at P.
5. At the surface, the field is one-fourth as strong as at P.
6. None of the above.

At surface field is 9x as strong as at P.

-stronger closer to source

$E(r=0.10\,m) = \dfrac{kQ}{(0.10\ m)^2}$ and $E(r=0.30\,m) = \dfrac{kQ}{(0.30\ m)^2}$

$= \dfrac{kQ}{[3(0.10\,m)]^2}$

$= \dfrac{kQ}{9(0.10\,m)^2}$

If the van de Graaff generator dome has lost 1 pmol (1×10^{-12} mol) of electrons, how strong is the force on an electron in an air molecule just above the dome's surface? → need find $\vec{E}\,(r = 0.10\text{ m})$

Charge on dome $= (1\times10^{-12}\text{ mol})(6.023\times10^{23}\ e^-/\text{mol})(1.609\times10^{-19}\text{ C})$ ← elem. charge of proton

$\cong 1\times10^{-7}\text{ C}$ (or $0.1\ \mu\text{C}$)

Electric field $\quad E = \dfrac{kQ}{r^2} = \dfrac{(9\times10^{9}\ \text{Nm}^2/\text{C}^2)(1\times10^{-7}\text{ C})}{(0.10\text{ m})^2} = 9\times10^{4}\ \text{N/C}$

"Breakdown field" of air — get sparks — is 3×10^{6} N/C

Force on 1e⁻

$$F_{\text{one } e^-} = q E_{\text{dome}} = (-1.609\times10^{-19}\text{ C})(9\times10^{4}\ \text{N/C}) = -1\times10^{-14}\text{ N}$$

Breakdown field means field which is strong enough that air molecules placed in that field are pulled apart into ions and electrons

- dome's charge → $\vec{E}_{\text{dome}}$ is strong enough to pull apart air molecules

Field line diagrams

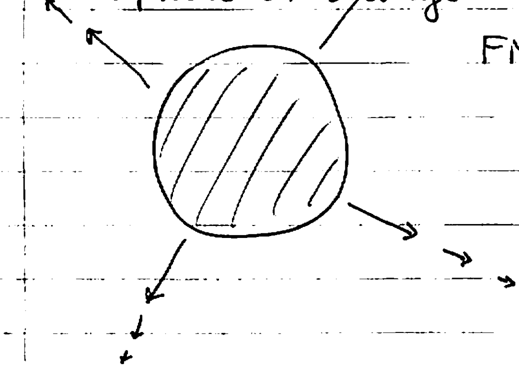
$\vec{E}$ fields are very 3D — vector at every point in space!

Another way to represent $\vec{E}$ visually is by drawing field lines: follow the field step by step, ~~so that each line shows the direction of the field along~~ (field is tangent to lines everywhere)

lines show direction of field; lengths don't mean anything

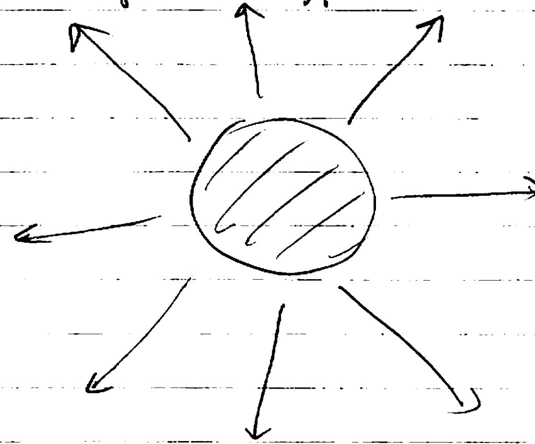
Example:

sphere of charge $+Q$

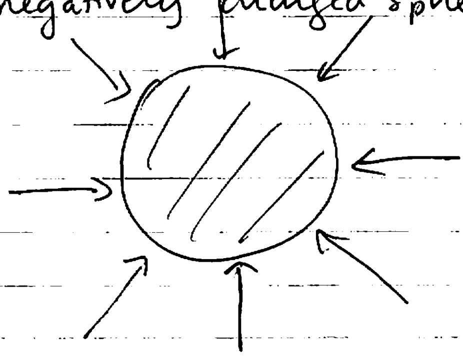


Field vectors (show Physlet also)/instead?)

Field lines



A negatively charged sphere:



Field lines, like field vectors, show us direction of force on a + charge placed in the field; force is opposite on ⊖

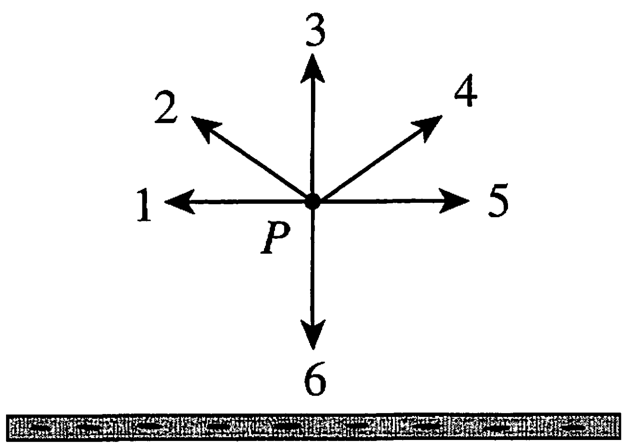
Properties of field lines we can observe from this:

1. Begin on ⊕, end on ⊖ (or can begin/end ∞ far away from any charges) — sources of field are charges
2. Strength of field $\propto$ how close together lines are (not exact)
3. Lines do not cross or touch

Next benchmark arrangement of charge: line of charge

Handout: STEP 1
Ask question
Remaining steps

Consider a short rod carrying a uniformly distributed negative charge.



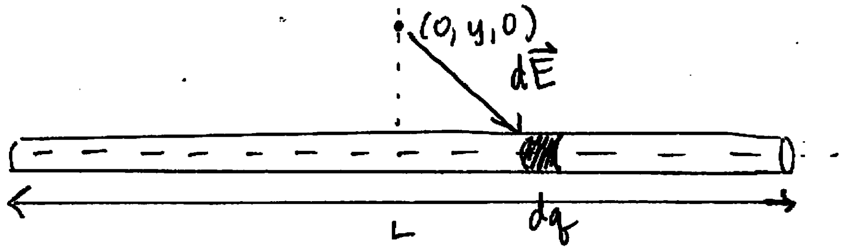
Which vector most closely represents the direction of the electric field at point P? (Due to whole rod)

1. Vector 1.
2. Vector 2.
3. Vector 3.
4. Vector 4.
5. Vector 5.
6. Vector 6. see STEP 2 on handout
7. The answer depends on the sign of the charge on the 'test' particle

Calculating the field of a uniformly negatively charged rod

STEP 1

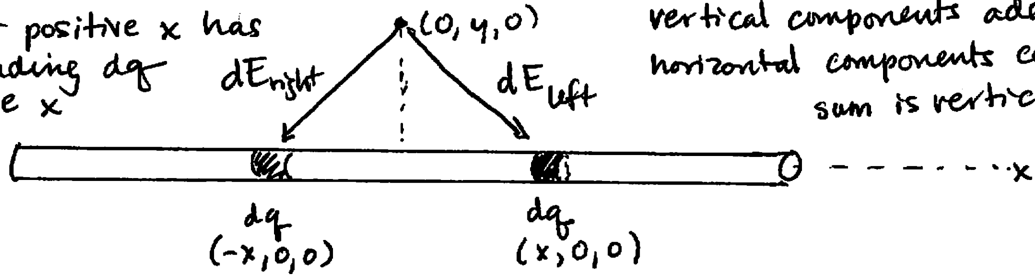
Identify location (x, y, z) where $\vec{E}$ is to be found ("field point")
Break rod into pieces of charge dq that can be treated like point charges
Draw vector for electric field $d\vec{E}(x,y,z)$ of piece dq



STEP 2

Identify any symmetries that lead to components of $\vec{E}$ canceling at the field point

Every dq at positive x has a corresponding dq at negative x



vertical components add
horizontal components cancel
sum is vertically down

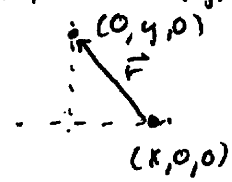
STEP 3

Write a mathematical expression for $d\vec{E}(x,y,z)$ using the electric field of a point charge
If appropriate, find component of $d\vec{E}$ that will contribute to total $\vec{E}$

$$d\vec{E} = \frac{kdq}{r^2}\hat{r} = \frac{k\,dq}{(x^2+y^2)}\,\frac{(-x\hat{\imath}+y\hat{\jmath})}{(x^2+y^2)^{1/2}}$$

$\vec{r}$ = vector from $(x,0,0)$ to $(0,y,0)$
$= -x\hat{\imath}+y\hat{\jmath}$

$$\hat{r} = \frac{-x\hat{\imath}+y\hat{\jmath}}{\sqrt{x^2+y^2}}$$



r is distance from dq to field point (x, y, z)

$\hat{r}$ is a unit vector pointing from dq to the field point

component of $d\vec{E}$ to integrate:

y components add, x-components cancel

$\Rightarrow$ integrate $dE_y = \dfrac{kdq}{(x^2+y^2)^{3/2}}\,y\hat{\jmath}$ at $(0, y, 0)$

STEP 4: integrate:

variable of integration is x
convert dq to be in terms of dx

$dq = \lambda\, dx$

charge/length

Limits of integration: ends of rod $x = -L/2$ to $L/2$
(add labels to figure)

Done in detail on next page

You do this in lab
infinite rod $\rightarrow$ limits are $\pm\infty$

STEP 4: Integrate

Choose variable of integration and identify limits on that variable
Express dq in terms of variable of integration and charge density
Take constants out of integral
Evaluate

Variable of integration is dx because rod goes from $x = -\frac{L}{2}$ to $x = \frac{L}{2}$ along the x-axis

Want to convert dq to something we can integrate over:

$$dq = \underset{\uparrow \text{ charge/length}}{\lambda} dx = -\frac{Q}{L} dx$$

Limits of integration come from ends of rod:

so our integral is

$$\vec{E} = \int d\vec{E}_y = \int \frac{k\,dq}{(x^2+y^2)^{3/2}} y\hat{j} = \int_{x=-L/2}^{x=L/2} \frac{k(-Q/L)\,dx}{(x^2+y^2)^{3/2}} y\hat{j}$$

Take constants out: y is a constant for this integration — it is the location of the point where we find the field — doesn't change as we move along the rod in x

$$\Rightarrow \vec{E} = -\frac{kQy}{L}\hat{j}\int_{x=-L/2}^{L/2} \frac{dx}{(x^2+y^2)^{3/2}}$$

and look up this integral in a table

$$\int_{x=-L/2}^{L/2} \frac{dx}{(x^2+y^2)^{3/2}}$$

$$\int_{-L/2}^{L/2} \frac{dx}{(x^2+y^2)^{3/2}} = \left.\frac{x}{y^2\sqrt{x^2+y^2}}\right|_{x=-L/2}^{x=L/2} = \frac{L/2}{y^2\sqrt{\frac{L^2}{4}+y^2}} - \frac{-\frac{L}{2}}{y^2\sqrt{\left(-\frac{L}{2}\right)^2+y^2}} = \frac{L}{y^2\sqrt{\frac{L^2}{4}+y^2}}$$

constant!

So $$\vec{E} = -\frac{kQ}{L}\cancel{y}\hat{j}\,\frac{L}{y^{\cancel{2}}\sqrt{\frac{L^2}{4}+y^2}} = \frac{-k\left(\frac{Q}{L}\right)L}{y\sqrt{\frac{L^2}{4}+y^2}}\hat{j}$$

So our result for $\vec{E}$ of the rod of length L and charge $-Q$, at a distance y from the center of the rod, becomes

$$\vec{E}(0,y,0) = \frac{k\left(-Q/L\right)}{y} \frac{L}{\sqrt{\frac{L^2}{4}+y^2}} \hat{j}$$

If the rod is very long compared to the distance y ($L \gg y$), then

$$\sqrt{\frac{L^2}{4}+y^2} \approx \sqrt{\frac{L^2}{4}} = \frac{L}{2}$$ because y^2 is much smaller than $L^2/4$ and can be ignored

Then our expression simplifies to

$$\vec{E}(0,y,0) \approx \frac{k\left(-Q/L\right)}{y} \frac{L}{\left(L/2\right)} \hat{j} = \frac{2k\left(-Q/L\right)}{y} \hat{j}$$

Usually we leave the charge per length symbolized as λ:

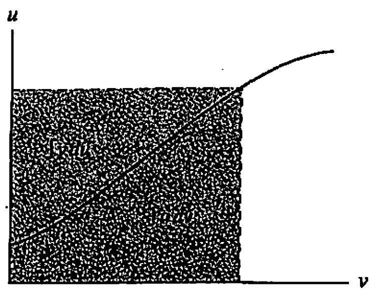
$$\vec{E}(0,y,0) = \frac{2k\lambda}{y} \hat{j}$$ (remember in this case $\lambda = -Q/L$ and so it's $\ominus$ in this case)

Using the other constant ϵ_0: substitute $\frac{1}{4\pi\epsilon_0}$ for k

$$\Rightarrow \vec{E}(0,y,0) = 2\left(\frac{1}{4\pi\epsilon_0}\right)\frac{\lambda}{y}\hat{j} = \frac{\lambda}{2\pi\epsilon_0 y}\hat{j}$$

This is our expression for $\vec{E}$ when the rod is much longer than the distance y.

Also, once $L \gg y$, this expression will be almost true if we are near the center.



2. Integration by parts

The quantity $\int u\,dv$ is the area under the curve of u as a function of v between specified limits. In the figure, that area can also be expressed as the area of the rectangle shown minus the area under the curve of v as a function of u. Mathematically, this relation among areas may be expressed as a relation among integrals:

$$\int u\,dv = uv - \int v\,du \qquad \text{(integration by parts)}$$

This expression may often be used to transform complicated integrals into simpler ones.

Example

Evaluate $\int x \cos x\,dx$. Here let $u = x$, so $du = dx$. Then $dv = \cos x\,dx$, so we have $v = \int dv = \int \cos x\,dx = \sin x$. Integrating by parts then gives

$$\int x \cos x\,dx = (x)(\sin x) - \int \sin x\,dx = x \sin x + \cos x$$

where the + sign arises because $\int \sin x\,dx = -\cos x$.

Table of Integrals

More extensive tables are available in many mathematical and scientific handbooks; see, for example, ***Handbook of Chemistry and Physics*** (Chemical Rubber Co.) or Dwight, ***Tables of Integrals and Other Mathematical Data*** (Macmillan). Some math software, including *Mathematica* and *Maple*, can also evaluate integrals symbolically. Wolfram Research provides *Mathematica*-based integration at *http://integrals.wolfram.com.*

In the expressions below, a and b are constants. An arbitrary constant of integration may be added to the right-hand side.

$$\int e^{ax}\,dx = \frac{e^{ax}}{a}$$

$$\int \sin ax\,dx = -\frac{\cos ax}{a}$$

$$\int \cos ax\,dx = \frac{\sin ax}{a}$$

$$\int \tan ax\,dx = -\frac{1}{a}\ln(\cos ax)$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int x \sin ax\,dx = \frac{1}{a^2}\sin ax - \frac{1}{a}x\cos ax$$

$$\int x \cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{1}{a}x\sin ax$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x + \sqrt{x^2 \pm a^2}\right)$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{x\,dx}{\sqrt{a^2 - x^2}} = -\sqrt{a^2 - x^2}$$

$$\int \frac{x\,dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

$$\int \frac{dx}{(x^2 \pm a^2)^{3/2}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$$

This is the integral to use, with the + sign

$$\int x e^{ax}\,dx = \frac{e^{ax}}{a^2}(ax - 1)$$

$$\int x^2 e^{ax}\,dx = \frac{x^2 e^{ax}}{a} - \frac{2}{a}\left[\frac{e^{ax}}{a^2}(ax - 1)\right]$$

$$\int \frac{dx}{a + bx} = \frac{1}{b}\ln(a + bx)$$

$$\int \frac{dx}{(a + bx)^2} = -\frac{1}{b(a + bx)}$$

$$\int \ln ax\,dx = x\ln ax - x$$

Field of an infinite line of charge

If the line of charge is very long compared to the distance y at which we want $\vec{E}$, we can use the limit of this result for $L >> y$. That result is

$$\vec{E}(0,y,0) = \frac{2k\lambda}{y}\hat{j}$$

or, rewritten using the other constant ϵ_0, $k = \frac{1}{4\pi\epsilon_0}$

$$\vec{E}(0,y,0) = \frac{\lambda}{2\pi\epsilon_0 y}\hat{j}$$

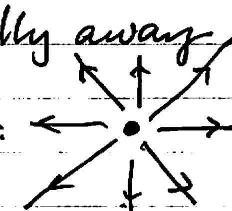
What about other directions away from the line? Any direction gives the same result as long as the distance r is measured perpendicular to the line

So defining $\vec{r}$ to be perpendicular to the line

$$\vec{E}(r) = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}$$

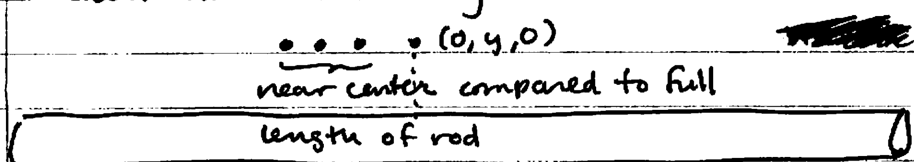
where $\hat{r}$ points radially away from the line

sketch of $\vec{E}$ field if λ is ⊕:



~~In cross section the electric field~~

In addition, we derived this at the center of the rod — but as long as we're near the center, the field will be basically the same. The cancellation of the horizontal components will basically apply still, and the summing of the vertical components won't change much.



So, we can use this result for the field of a long wire whenever we are

(1) near the middle (not near the ends)

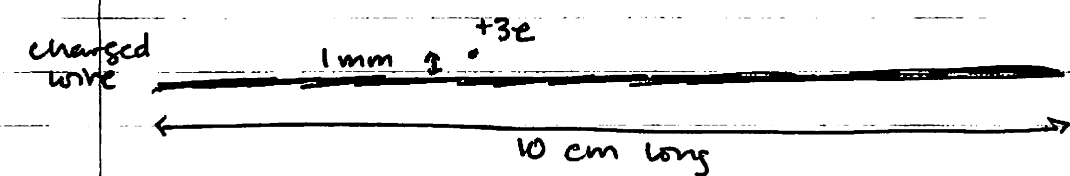
(2) a distance r away that is small compared to the length of the wire.

In cross-section, field of wire points radially away as shown above if λ is ⊕, radially toward wire if λ is ⊖

Example:

Find the force on an ion with charge $+3e$ that is 1mm away from a charged wire with total charge $-30\,pC$. The wire is 10 cm long and the ion is near the middle of the wire.

We can use the previous result because the wire is long compared to the distance the ion is from it:



$$\vec{E}_{wire}(r) = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}$$

We have $\lambda = \dfrac{-30\,pC}{10\,cm} = \dfrac{-30\times10^{-12}\,C}{0.10\,m} = -3.0\times10^{-10}\,C/m$

$\epsilon_0 = 8.85\times10^{-12}\,C^2/Nm^2$

$r = 0.001\,m$

$$\Rightarrow \vec{E}_{wire} = \frac{(-3.0\times10^{-10}\,C/m)}{2\pi(8.85\times10^{-12}\,C^2/Nm^2)(0.001\,m)} = (-5.4\times10^{3}\,N/C)\hat{r}$$

points radially in

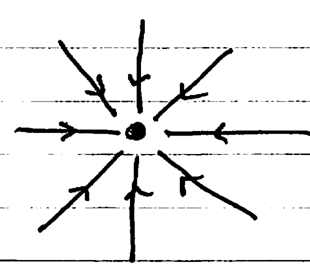
Therefore force on ion is

$$\vec{F}_{wire\ on\ ion} = q_{ion}\vec{E}_{wire} = (+3)(1.609\times10^{-19}\,C)(-5.4\times10^{3}\,N/C)\hat{r}$$

$$\vec{F}_{wire\ on\ ion} = (-2.6\times10^{-15}\,N)\hat{r}$$

also points radially in

$\boxed{\text{Cross-sectional views of } \vec{E}_{wire}:}$



force is same direction as $\vec{E}$ because ion's charge is $+$