Announcements 2/11/10 (posted with class notes)

Posted on web site: schedule of SA sessions, review session, and office hours
Posted on web site: PS 4 (one or two more problems coming) detailed information about upcoming midterm, exams from Physics 4L 2008.

We do have class both Tuesday and Thursday the week of the exam, but there is no lab that week to give you some extra time.

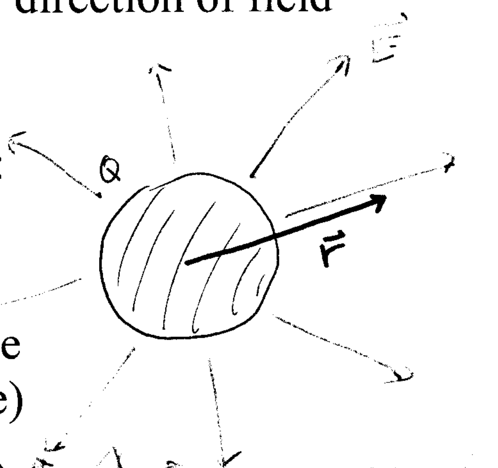
Lab next week:
There is a prelab assignment, but Monday and Tuesday lab attendees don't need to do warmups 4, 5, 6 before lab; just do warmups 1,2, 3, read, and write a summary paragraph. Bring your textbook to lab.

Reading for next week:
Next Tuesday: **22.2**, omitting "Continuous charge distributions" and Ex. 22.6 and 22.7, and **22.3** only up through "Got it?" 22.6
Most likely, no additional reading for next Thursday.

Key ideas from last time

Worked out electric fields of sphere and very long rod
Field line representation: draw lines that show direction of field everywhere, spread out as field gets weaker

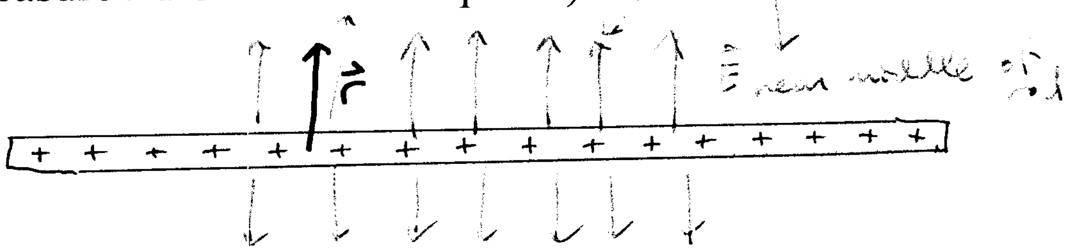
Sphere (either solid or hollow spherical shell):



$$\vec{E}(r) = \frac{kQ}{r^2}\hat{r} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

Q = total charge of sphere, r outside sphere
($r > a$ and measure r from center of sphere)

Very long rod:



$$\vec{E}(r) = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$$

$\lambda = Q_{rod}/L$ and $\hat{r}$ points perpendicularly out from rod
r measured from axis of rod

Polarization:
in an electric field, electron density in most materials shifts a little bit to produce a separation of charge — a dipole moment

J.J.

2/11/2010

Three items for today:

(1) Last of benchmark electric fields: sheet of charge

(2) Forces & torques on dipoles in $\vec{E}$

(3) Begin potential

Sheet of charge

Imagine a very large, flat sheet covered uniformly with charge

~~Area density~~ Charge per area $\sigma = Q/A$ is constant

The field of this is uniform & points $\perp$ to sheet

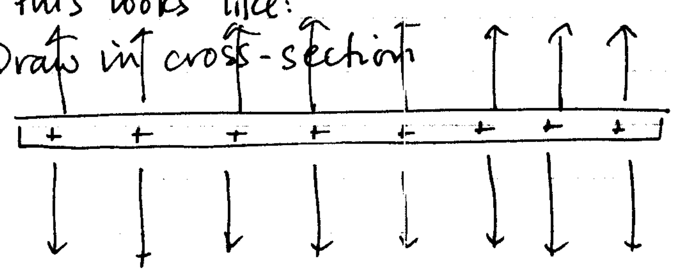
Magnitude is

$$E = 2\pi k\sigma = \frac{\sigma}{2\epsilon_0}$$ depending on whether you use k or ϵ_0 (recall $k = \frac{1}{4\pi\epsilon_0}$)

Going forward I'm going to just use ϵ_0, you can use either

What this looks like:

Draw in cross-section



and opposite if sheet is ⊖

handout has 3D picture

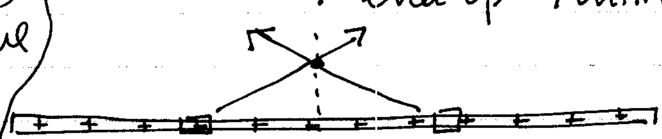
show styrofoam

Key: does not depend on distance, if you're close to the sheet!

Why? – exactly true if sheets are infinite in size

– same idea as rod: if you're near the middle, charges to either side exert opposing horizontal components of $\vec{E}$ but same direction vertical

$\Rightarrow$ end up w/entirely vertical



– can be near the middle and this is still basically true

– putting many lines next to each other $\rightarrow$ no distance dep.

Retread

Rod: this result is exactly true for the center of ∞ long rod

It is approx true near center of long but not ∞ rod

no such thing as ∞ ...

after all membr example

This will be true if you are a distance y away from the sheet that is much smaller than the size L of the sheet

BUT: if you are far away compared to size of sheet doesn't work at all!

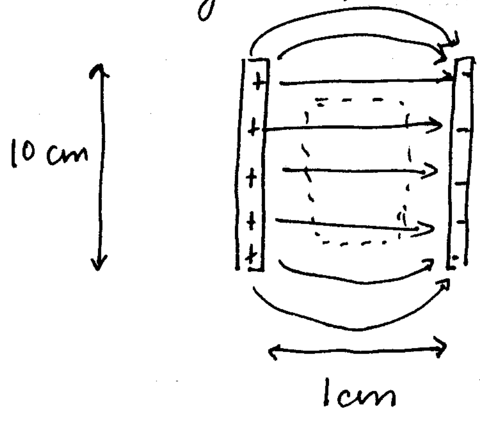
11:45 up to here

$\boxed{\text{CT:}}$ two sheets
Field between is σ/ϵ_0
outside is zero!

This is actually our most useful field result.

(1) This is how you make a uniform $\vec{E}$ in real life. Get two flat metal sheets, give them opposite charges: in between, near the middle, field is uniform. Starts to bend near edges, but stay in middle and you're OK!

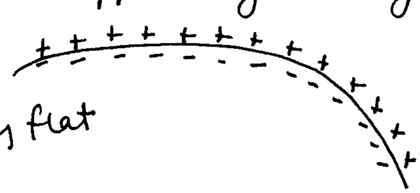
(Draw better to scale)



You'll measure a model of this in Lab next week.

If plates are big compared to separation — say 10 cm × 10 cm square plates, 1 cm apart — then ~~most~~ a lot of the space inside is close to the middle of the plates again approximate as ∞ plates

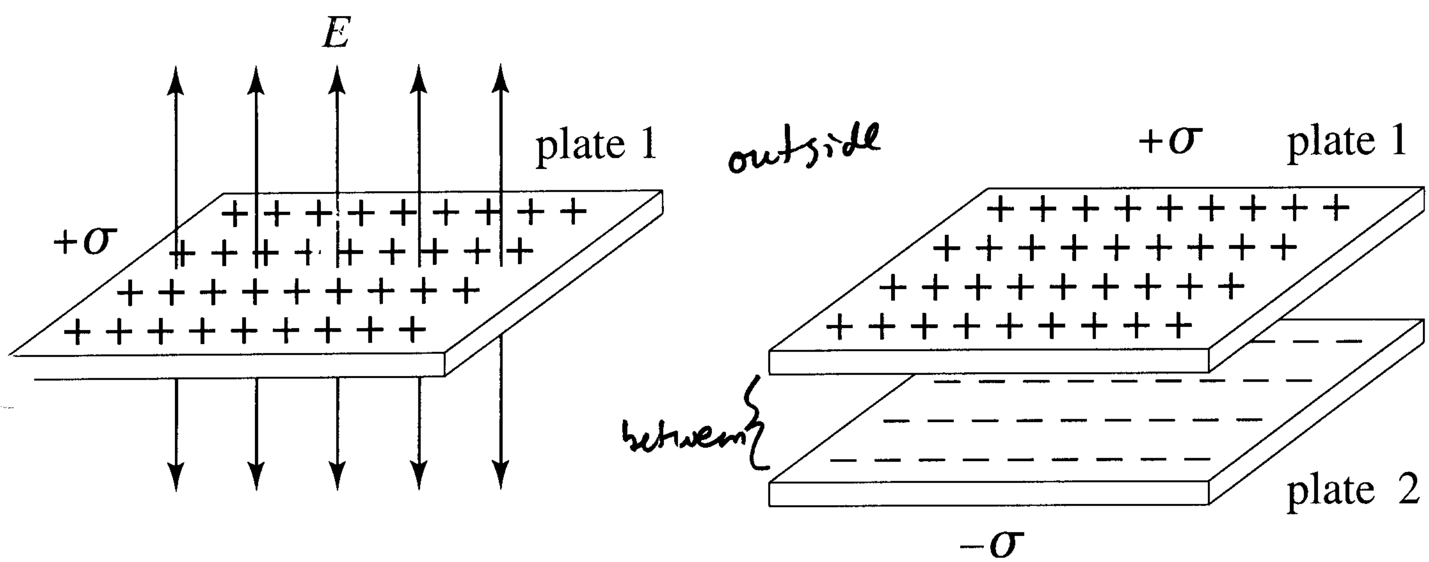
(2) Cell membrane is remarkably well ~~modelled~~ approximated as two oppositely charged sheets of charge.



Draw more carefully: very flat

Just as the Earth is roughly spherical, but it's so big that it looks flat to us:
cell membrane is only a few nm thick (10^{-9} m); cell is several μm (10^{-6}) overall size

The electric charge per unit area is $+\sigma$ for plate 1 and $-\sigma$ for plate 2. The magnitude of the electric field associated with plate 1 is $\sigma / 2\epsilon_o$, and the electric field lines for this plate are as shown. When the two are placed parallel to one another, the magnitude of the electric field is



1. σ / ϵ_o between, 0 outside.
2. σ / ϵ_o between, $\pm\sigma / 2\epsilon_o$ outside.
3. zero both between and outside.
4. $\pm\sigma / 2\epsilon_o$ both between and outside.
5. none of the above.

So we can very well treat $\vec{E}$ in the membrane, between the two sheets of charge, as constant:

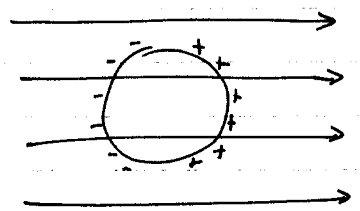
two sheets are only 7 nm apart

size of sheets is hundreds of nm

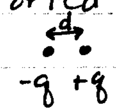
More importantly: $\vec{E}$ outside the region between the two charge layers is very small — strong fields in between the two layers do not really escape outside.

Forces on ~~[illegible]~~ dipoles in electric fields

Last time briefly introduced idea of polarizing objects in $\vec{E}$

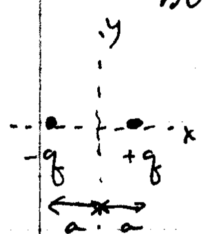


Imagine putting neutral sphere in $\vec{E}$
$\ominus$ ~~[illegible]~~ feels force opposite $\vec{E}$ - shifts
$\oplus$ is left behind

Said this was because molecules distort: e^- clouds pulled away from being centered
Each molecule has distorted to 
Can model as a dipole  $\pm q$ separated by d
with the $-q$ at the center of the electron cloud

Called "induced dipole" b/c goes away if remove $\vec{E}$

Define dipole moment $\vec{p} = qd\,\hat{r}_{-+}$ points from − to +
tells us how much charge separation there is
key is not how much charge or how much separation, but their product:



look at $\vec{E}_{dipole}$ found previously $\vec{E}_{dipole}(0,y,0) = -\frac{2kqa}{(a^2+y^2)^{3/2}}\hat{\imath} \approx -\frac{2kqa}{y^3}\hat{\imath}$

Dipole moment here is $\vec{p} = \underset{q}{q}\underbrace{(2a)}_{d}\underset{\hat{r}_{-+}}{\hat{\imath}}$

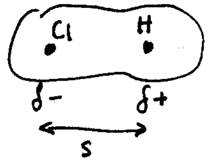
?

$\Rightarrow \vec{E}_{dipole}(0,y,0) \approx -\frac{kp}{y^3}\hat{\imath}$ Similarly field on axis $\sim \frac{kp}{x^3}$

So $\vec{E}_{dipole}$ depends on p
Likewise forces on dipoles turn out to all depend on p;
Look back at your result from Prob 62: can rewrite in terms of dipole moment

$\boxed{\text{CT}}$ Direction of induced $\vec{p}$? Always $\parallel$ to $\vec{E}$ that causes it

Some molecules have <u>permanent</u> dipole moments :
i.e. HCl molecule : e^- pulled closer to Cl nucleus than H nucleus



often notated in chemistry as having partial ⊕ & partial ⊖ — effective charge $\delta +$ at H end, $\delta -$ at Cl end

Dipole moment here would be $p = \delta s$
Typically these partial charges are tiny : for HCl $\delta = 0.16e$
$s = 0.127$ nm ← 10^{-9} m

$\Rightarrow p = 3.43 \times 10^{-30}$ C·m

Best known permanent dipole moment : H_2O

<u>Forces (~~and torques~~) on dipoles in $\vec{E}$</u>
~~Under what circumstances~~ does a dipole feel a force ?
12:10 ~~CT~~ In what sort of $\vec{E}$
To answer this : think about vector sum of forces on two ends of dipole

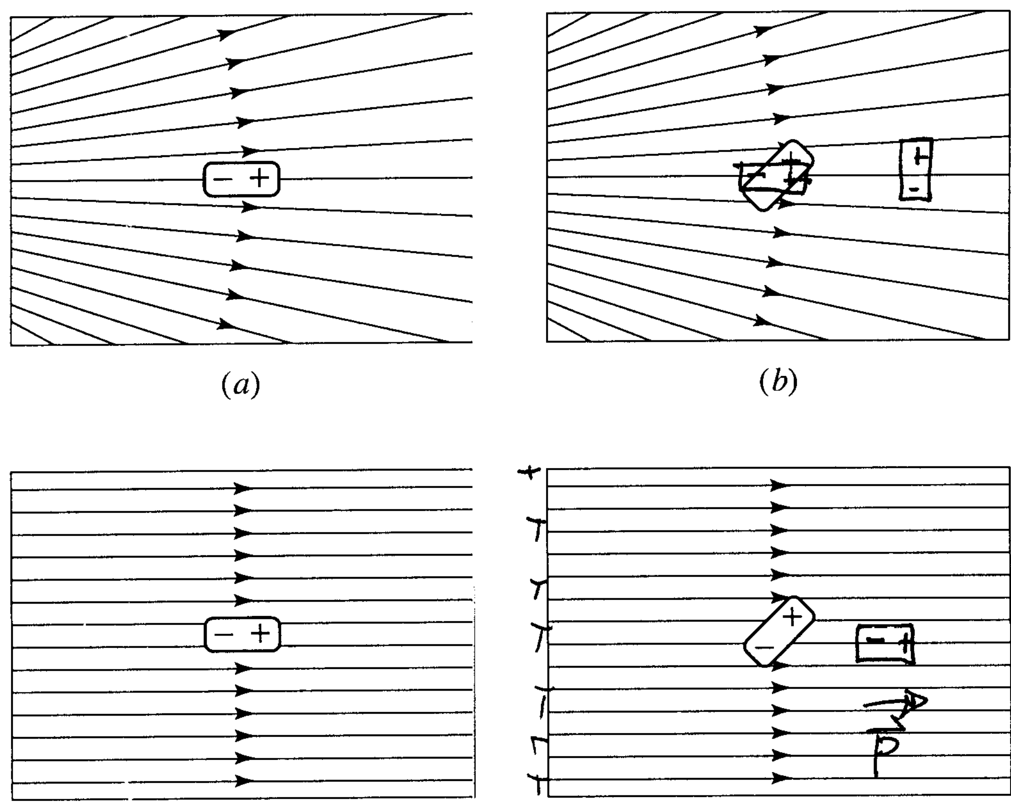
CT Net force ?
zero if $\vec{E}$ is uniform
(but if $\vec{p}$ is at an angle to $\vec{E}$, it will turn — a torque)

nonzero force if $\vec{E}$ is nonuniform!
then find force by summing forces on ends, as you did on HW

~~To calculate force you'll do~~
you have another such problem on HW coming up : force on dipole in field of ion — note that in this case force is <u>stronger</u> than in case of dipole-dipole force

van der Waals force : between two <u>induced</u> dipoles

An electrically neutral dipole is placed in an external field. In which situation(s) is the net force on the dipole zero?



(*a*) (*b*) (*c*) (*d*)

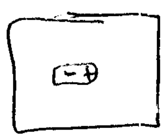
both move left

$\vec{E}$ is uniform

(field lines ∥)

forces on ends are equal mag, opp direction

rotate so $\vec{p}$ in same dir as $\vec{E}$



1. (*a*)
2. (*c*)
3. (*b*) and (*d*)
4. (*a*) and (*c*)
5. (*c*) and (*d*)
6. some other combination
7. none of the above

12:15

Electric potential difference

Now we're going to use these electric fields that we have painstakingly found to work out the expression for electric potential energy.

We want to be able to solve problems using electric potential energy rather than forces because energy is a scalar while force is a vector, so generally it's much easier to solve problems that don't require vectors.

Think again about gravitational potential energy. Consider putting yourself at the top of a frictionless hill on a sled and sliding down — you could find your speed at the bottom by determining the force that was exerted on you and the sled, or you could simply calculate the change in potential energy to find the change in kinetic energy. (Hopefully all of you have had the chance to do some experiments like this recently ... presumably without bothering to record or analyze the data!)

We want to do the same here — we want to find a way to calculate electric potential energy and use that instead of electric fields and forces.

In the lab this week you've already started working through a qualitative picture. Those of you who have already been to lab have seen that electric potential energy decreases as a charged particle has its kinetic energy increased by an electrical force, just like gravitational potential energy decreases as a massive object has its kinetic energy increased by the gravitational force.

Gravity: B — $U^G_B + K_B$

$\Delta U^G_{BA} = U_B - U_A$

$\Delta K = K_B - K_A$

A — $U^G_A + K_A$

$\Delta U^G + \Delta K = 0$ if only force is gravity

So if K increases U^G decreases

How can we find a quantitative expression for potential energy? We find the work done moving between two points when the electrical force is the only force acting on a charged particle. The work-energy theorem tells us that if the electrical force is the only force, then the charged particle's kinetic energy will change by the amount of the work done. Therefore this must also equal the amount of potential energy lost by the charged particle, or the opposite of the change in potential energy.

$$W_E = \Delta K = -\Delta U^E_{AB}$$

(ΔU^E_{AB} is the change in electric potential energy going from A to B)

So, let's calculate the work done by the electric force and this will give us the corresponding change in potential energy.

However, once again we'd like to have a way to understand the effect of the sources of the electric field that is separate from the amount and sign of the charge feeling the field. Although all masses feel the same direction force from the Earth, different signs of charge feel different directions of force from a source charge. We defined the electric field, as the force per charge, to let us characterize the effect that the sources would have on a positive charge feeling the force, and then we know that the force on a negative charge is the reverse.

In the same way we define a quantity that is *change in electric potential energy per charge*

$\Delta V_{AB} = \dfrac{\Delta U_{AB}^{E}}{q}$ where once again q refers to the charge feeling the effect of the electric field, not the source charges that are producing the electric field

Confusingly enough, it's called *electric potential difference.* NOT the same as potential energy! (You would think physicists could be more creative! At least electric field and force have different names.....)

What is this analogous to? Think again about gravity:

Near the surface of the earth, change in gravitational potential energy is

$$\Delta U_{AB}^{G} = mg\Delta h$$

So the analogous quantity "gravitational potential difference" would be

$$\frac{\Delta U_{AB}^{G}}{m} = g\Delta h$$

So let's calculate this. We said that the change in potential energy was the opposite of the work done by the electrical force. For constant electric field we can find the work as the dot product of force and displacement:

$$W^{E} = \vec{F} \bullet \Delta\vec{r}_{AB} = q\vec{E} \bullet \Delta\vec{r}_{AB} = qE\Delta r_{AB}\cos\theta$$

Therefore for constant electric field

$$\Delta V_{AB} = \frac{\Delta U_{AB}^{E}}{q} = \frac{-W^{E}}{q} = -\vec{E} \bullet \Delta\vec{r}_{AB} = -E\Delta r_{AB}\cos\theta$$

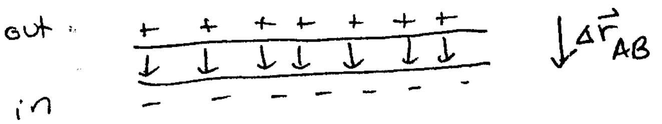
This says that the potential difference between two points (in a constant electric field) is given by the strength of the field multiplied by the component of the vector from one point to the other that is parallel to the electric field.

Potential differences are what we measure — units are volts! Use measured voltage across a cell membrane to work out the electric field in the membrane:

CT: sign

Problem: strength of E

$\boxed{\text{CT}}$



$$\Delta V_{AB} = -\vec{E}\circ\Delta\vec{r}_{AB}$$

$\vec{E}$ and $\Delta\vec{r}_{AB}$ in same direction $\Rightarrow$ $\vec{E}\circ\Delta\vec{r}_{AB} = E\Delta r_{AB}$ $(+)$

so ΔV_{AB} $(-)$ ($(+)$ chg loses energy moving out $\rightarrow$ in)

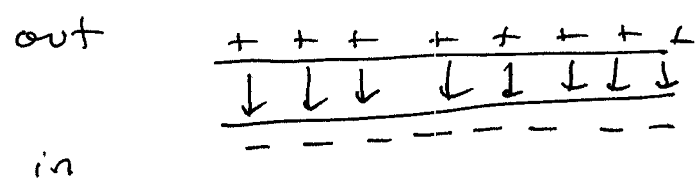
Strength of $\vec{E}$:

$$\Delta V_{AB} = -E\Delta r_{AB} \Rightarrow E = -\frac{\Delta V_{AB}}{\Delta r_{AD}} = -\frac{\overset{-0.070\,V}{-70\,mV}}{7\times10^{-9}\,m} = 1\times10^{7}\,N/C \; !!$$

If the inside of a cell membrane is negatively charged and the outside is positively charged, the potential difference from the outside to the inside is:

$\Delta V_{out\ in}$

1. negative
2. positive
3. need more information



moving in direction of $\vec{E}$

$\oplus$ particle gains K

loses U_E

$\rightarrow \ominus\ \Delta V_{out\ in}$