Announcements 4/13/10

Handout: PS 11

Reading:
Today: 27.6 (skipping 27.4 -27.5), qualitatively only; 29.4 up to "Gauss's Law", 29.5; if you're rusty on waves, review 14.1 and 14.2.
Thursday: 29.6-29.8

Review of mathematics of sinusoidal waves Thurs evening

Key ideas from last time

Continued to use Faraday's Law: $\mathcal{E}_{\text{ind}} = -\dfrac{d\Phi_B}{dt}$

Φ_B is calculated from external field, NOT B_{ind} of induced current

Lenz's Law: direction of induced current opposes the change that created it

if flux of external B is increasing, B_{ind} of induced current is opposite B, and vice versa

work done to move conductor goes into I^2R heating of conductor

Final application (today) of moving conductor: eddy currents

J.J.

4/13/10

Today:

1. Eddy currents
2. Example of induced current from time-dependent $\vec{B}$

~~3.~~

3. Chemical shifts in NMR/diamagnetism
4. Induced $\vec{E}$ and $\vec{B}$ $\Rightarrow$ electromagnetic waves

changing $\vec{B} \Rightarrow$ induced $\vec{E}$ that loops around $\frac{d\vec{B}}{dt}$

<u>NOT</u> a conservative field

changing $\vec{E} \Rightarrow$ induced $\vec{B}$ that loops around $\frac{d\vec{E}}{dt}$

$\Rightarrow$ traveling patterns of $\vec{E}$ and $\vec{B}$!

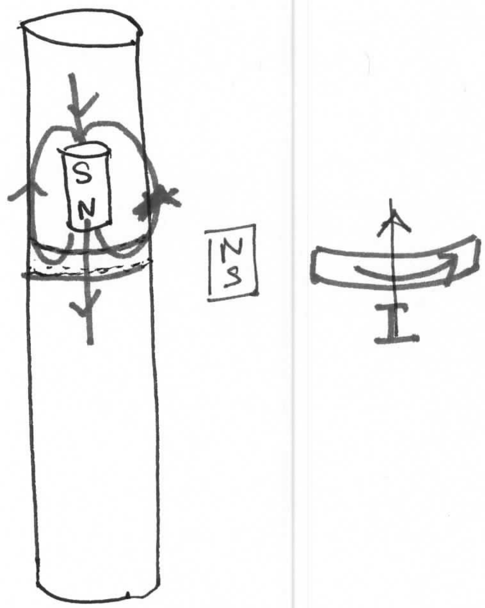
Eddy currents

Even if you don't have a circuit or a loop, ^mobile charges in any conductor will feel forces if it moves in nonuniform $\vec{B}$ — or if source of $\vec{B}$ moves!

Basis of our dropping/rolling magnet demos:
as magnet falls, current patterns induced that cause forces resisting the motion?

Why resist motion? conservation of energy again
as magnet falls, induced current flows in copper – I^2R power dissipation heats up copper
energy must come from somewhere – U_{grav} of magnet converted to heat as well as KE
(just like pulling bar)

A strong permanent magnet is dropped through a vertical conducting tube with the magnet's north pole down. As the magnet falls, current flows in rings around the tube axis.



magnet's Φ_B through loop is increasing
→ current gives B_{ind} upward (opposite)

Force must be up to slow magnet
→ induced current must have $\vec{\mu}$ opposite $\vec{\mu}$ of magnet

As the magnet falls, the rings of current below the magnet flow

1. clockwise as viewed from above
2. counterclockwise as viewed from above

Chemical shifts in NMR

NMR spectroscopy: many forms

simplest: measure energy absorbed in flipping proton (put sample in strong B_0)

(nuclear) spins from parallel to antiparallel

$\Delta U = 2\mu B_{total}$ so measuring ΔU = measuring B_{total}

characterize non magnetic molecules (no net $\vec{\mu}$ of electron spins)

Key: B_{total} is not exactly same as the applied $\vec{B}_0$ — for reasons of induction

Although molecule's electron spins don't give net $\vec{\mu}$, electron orbital motion around nucleus is like a current

When molecule is placed in $\vec{B}_0$, electron orbital motion changes slightly to oppose change in flux through the orbit — like jumping ring demo

→ induced $\vec{\mu}$ that opposes $\vec{B}_0$

→ induced $\vec{B}_{ind}$ that slightly offsets $\vec{B}_0$

Take out PS - look @ figure

The amount of this induced $\vec{B}_{ind}$ determines

$\vec{B}_{total} = \vec{B}_0 + \vec{B}_{ind}$ in vicinity of each proton

and depends on details of electronic structure ~~of molecule~~

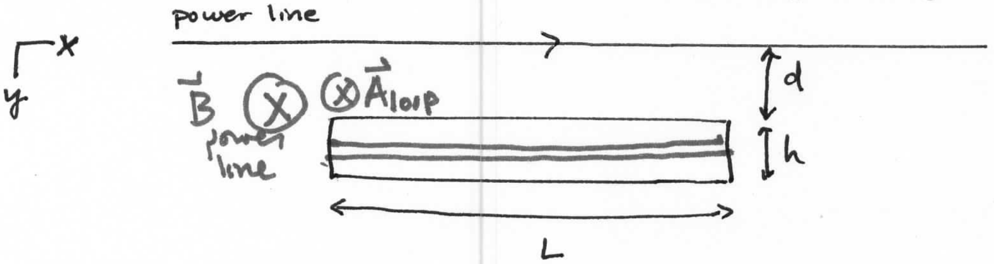
→ different chemical species have differing magnitudes B_{total} & hence different energies they absorb; different protons w/in molecules

The ~~difference~~ NMR spectrum thus indicates how the neighboring electron orbitals respond to the applied $\vec{B}_0$

Problem on your problem set: you work out direction of induced electron current

$\boxed{\text{DIAMAGNETISM}}$ (levitating frog)

An unscrupulous farmer rigs up a single rectangular wire loop directly beneath a high-voltage power line on his farm in order to get some electric power for free. The power line carries a current oscillating at 60 Hz with peak value 10 kA. The loop is 0.50 m high vertically, and the upper side of the loop is 2.0 m beneath the power line. If the farmer wants to have a peak emf of 170 V to be comparable to household power, how long must the loop be horizontally? How could the farmer make the loop smaller and still obtain the desired peak voltage?



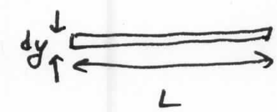
$I = I_0 \cos \omega t$
$= I_0 \cos(2\pi f t)$

We need to find L that will give a particular peak voltage (emf).
Faraday's Law tells us how much induced emf we get:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$ so we need to find $\Phi_B = N\int \vec{B}\cdot d\vec{A}$ through loop

$\vec{B}_{\text{power line}}$ is into page in plane of loop (when $\cos \omega t$ is ⊕) so choose $\vec{A}_{\text{loop}}$ to also be into page. $\Rightarrow$ $\vec{B}\cdot d\vec{A} = BdA\cos 0^\circ = BdA$

Divide loop into horizontal strips of width L and height dy



$$\Rightarrow dA = L\,dy \quad \text{and in that strip, } B = \frac{\mu_0 I}{2\pi y}$$

$$\Rightarrow \Phi_B = N\int B\,dA = N\int_{y=d}^{y=d+h} \frac{\mu_0 I}{2\pi y} L\,dy$$

Use $$\int_a^b \frac{dy}{y} = \ln y \Big|_a^b = \ln b - \ln a = \ln\left(\frac{b}{a}\right)$$

$$\Rightarrow \Phi_B = \frac{N\mu_0 I L}{2\pi} \ln\left(\frac{d+h}{d}\right)$$

Now we need $\frac{d\Phi_B}{dt}$. Only I depends on time. $I = I_0 \cos \omega t$

$$\Rightarrow \frac{dI}{dt} = -\omega I_0 \sin \omega t = -2\pi f I_0 \sin(2\pi f t)$$

$$\Rightarrow \frac{d\Phi_B}{dt} = \frac{N\mu_0 L}{2\pi} \ln\left(\frac{d+h}{d}\right) \frac{dI}{dt} = -\frac{2\pi f I_0 N \mu_0 L}{2\pi} \ln\left(\frac{d+h}{d}\right) \sin(2\pi f t) = -\mathcal{E}$$

The peak value of the emf will be the value of $\mathcal{E} = -\frac{d\Phi_B}{dt}$ when $\sin(2\pi f t) = 1$. Canceling 2π's:

$$\Rightarrow \mathcal{E}_{peak} = f I_0 N \mu_0 L \ln\left(\frac{d+h}{d}\right)$$

Solve for L:

$$L = \frac{\mathcal{E}_{peak}}{f I_0 N \mu_0 \ln\left(\frac{d+h}{d}\right)}$$

Substitute values:

$d = 2.0\ m$ $\quad$ $\mathcal{E}_{peak} = 170\ V$

$h = 0.50\ m$ $\quad$ $I_0 = 10 \times 10^3\ A$

$N = 1$ $\quad$ $f = 60\ Hz$

$\Rightarrow L = 1000\ m$!!

How could we shorten the loop and still get desired peak $\mathcal{E}$?

- add more turns
- make loop h greater and/or move loop closer to power line (d smaller)

Induced $\vec{E}$ and $\vec{B}$

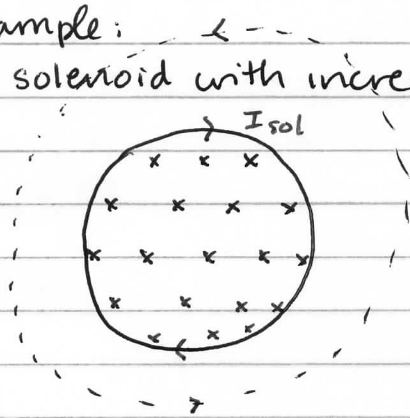
What causes $\mathcal{E} = -\frac{d\Phi_B}{dt}$ when the conductor is stationary but $\vec{B}$ is moving or changing in time?

must be an electric field – only $\vec{E}$ exerts forces on stationary charges

Turns out that whenever there exists a changing $\vec{B}$, it is accompanied by an induced electric field – even if there is no conductor present!

Example:

solenoid with increasing current $B_{sol} = \mu_0 n I_{sol}$



Draw w/colored chalk?

As I_{sol} increases, if there is a conducting loop where the dotted line is, get an induced current (which way? opposite I_{sol})

This current flows b/c there exists $\vec{E}_{ind}$ that also circles CCW around the solenoid

Even if the loop isn't there, there is still $\vec{E}_{ind}$: put a proton there and it feels a force!

Think of $\frac{dB}{dt}$ as being like a current – induced $\vec{E}$ circles around $\frac{dB}{dt}$

(can always work out direction of E_{ind} by imagining a circular current loop)

This is a fundamental property of electric and magnetic fields – whenever there exists $\frac{d\vec{B}}{dt}$, $\vec{E}_{ind}$ exists

[Mathematically: $\mathcal{E} = \oint_{\text{around loop}} \vec{E} \cdot d\vec{r}$ is work per charge done by $\vec{E}_{ind}$ (when conductor moves, supplied by $\vec{v} \times \vec{B}$)]

Note that now $\vec{E}_{ind}$ is <u>not</u> a conservative field

before $\int_A^B \vec{E} \circ d\vec{r}$ was path-independent

Now it matters which way you go around the circle

because $\vec{E}$ loops in a circle

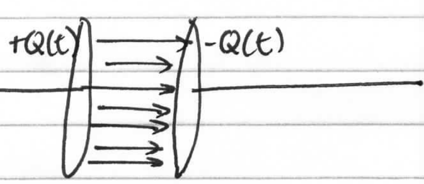
$\Delta V_{AB} = - \int \vec{E}_{ind} \circ d\vec{r}$ depends on path now

Changing $\vec{E} \Rightarrow$ induced $\vec{B}$

To complete the symmetry: ~~we~~ said that $\vec{E}$ and $\vec{B}$ are really manifestations of the same interaction just viewed from different perspectives – we'll discuss this the last day of class

So: what might you guess happens when you have a changing $\vec{E}$? Such as a charging capacitor:

Draw it vertically
show $\vec{B}$
leave room



Yes – $\frac{d\vec{E}}{dt} \Rightarrow \vec{B}_{ind}$

$\frac{d\vec{E}}{dt}$ acts exactly like a current – get $\vec{B}_{ind}$ circling around it just as it would around current $\left[\oint_{loop} \vec{B}_{ind} \cdot d\vec{r} = \mu_0 \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}\right]$

$\Rightarrow$ This is the basis of electromagnetic waves!

In the 19th century physicists had clues that light was a wave, because light was observed to form patterns similar to those seen by water waves

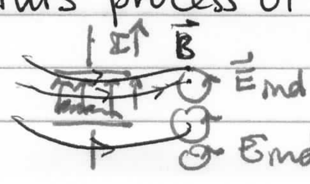
Question: wave of what?

show slinky
transverse
wave

mechanical wave: disturbance in a spring (or in water) that carries energy away from source ~~travels without changing its shape~~
– disturbance is self-reinforcing (slinky itself does not travel)

EM wave: a self-reinforcing pattern of $\vec{E}$ and $\vec{B}$ that travels away from its source
self-reinforcement relies on this process of induction of both $\vec{E}$ and $\vec{B}$

Back to capacitor:



Sinusoidal electromagnetic waves

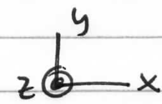
EM waves are transverse: because $\frac{d\vec{B}}{dt} \Rightarrow \vec{E}_{ind}$ circling around and $\frac{d\vec{E}}{dt} \Rightarrow \vec{B}_{ind}$ circling around, always have $\vec{B}$ and $\vec{E}$ $\perp$

(think how $\vec{B} \perp$ current for a wire); also $\perp$ to dir of travel

In EM waves, max $\vec{B}$ and max $\vec{E}$ coincide — again due to mutual induction

Simplest possible mathematical form of wave:

$$\vec{E}(x,t) = E_p \sin(kx - \omega t)\,\hat{j}$$
$$\vec{B}(x,t) = B_p \sin(kx - \omega t)\,\hat{k}$$



1. Transverse: $\vec{E} \perp \vec{B} \perp$ direction of travel
2. $\vec{E}, \vec{B}$ max at same time/location: "in phase" (same sin( ))
3. "Plane wave":
   $\vec{E}, \vec{B}$ same throughout a plane at a fixed x — depend only on x, not on y or z

How do we represent such a wave?
- "Snapshot" at a given t shows a single vector $\vec{E}$ and vector $\vec{B}$ at each x
  Those vectors indicate $\vec{E}, \vec{B}$ everywhere in the plane corresponding to that x
- as t progresses, pattern shifts along direction of travel

Show how fields move by drawing at t=0 and later t

How do you know travel direction?
$\sin(kx - \omega t)$ is moving in +x; $\sin(kx + \omega t)$ moves in −x
Also: due to induction, travels in direction of $\vec{E} \times \vec{B}$

$\vec{E}(x,t) = E_p \sin(kx - \omega t)\hat{j}$

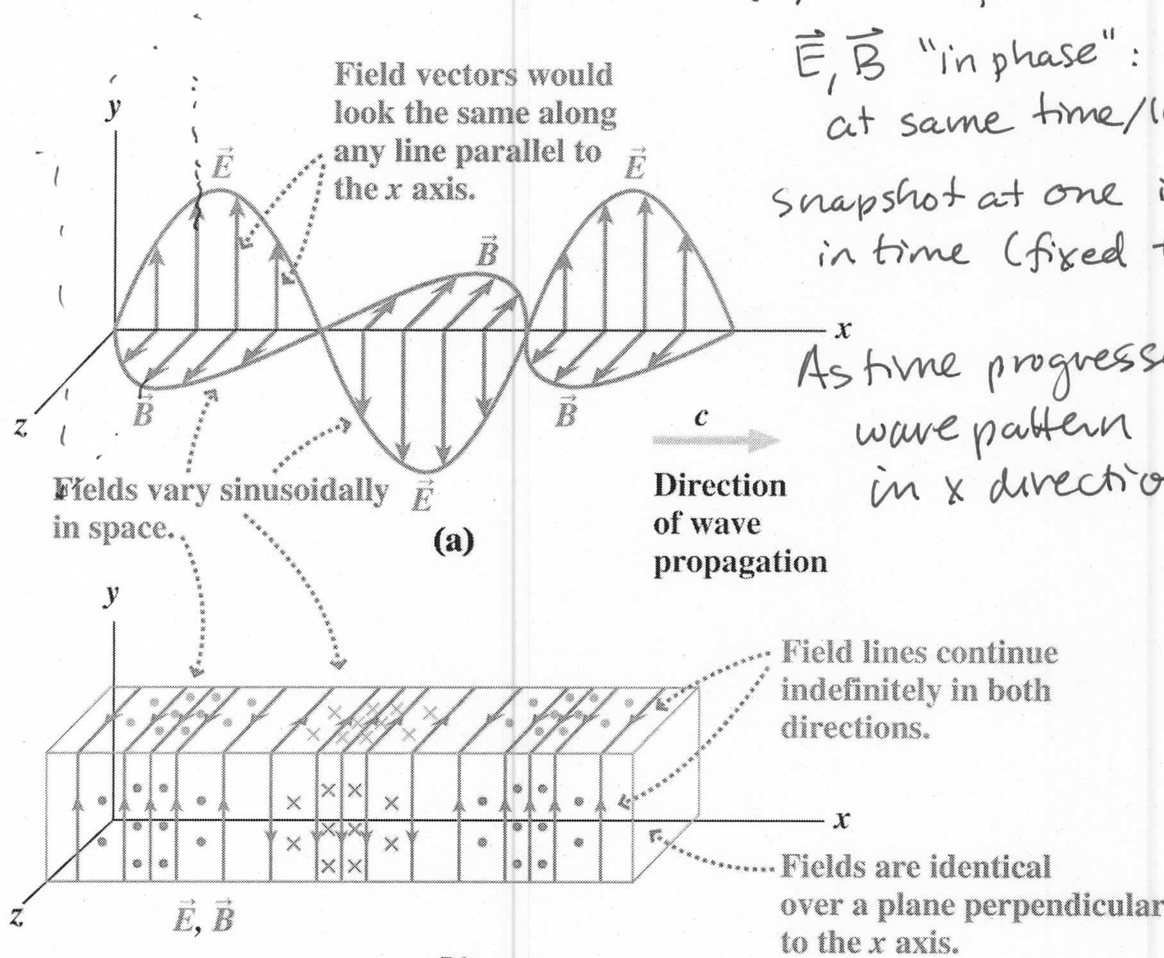
$\vec{B}(x,t) = B_p \sin(kx - \omega t)\hat{k}$

$\vec{E}, \vec{B}$ "in phase": max at same time/location

snapshot at one instant in time (fixed t)

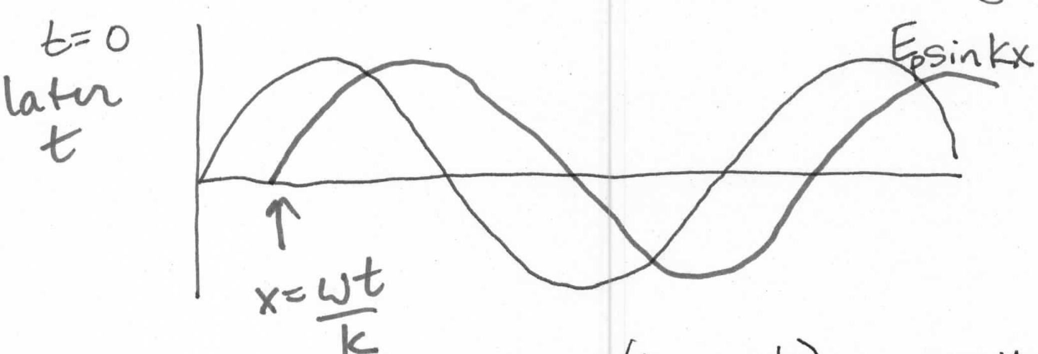
As time progresses, wave pattern moves in x direction

Electromagnetic wave: figure 29.3 from Wolfson



Which way does it move?

$\sin(kx - \omega t)$ describes moving in $+x$ direction



$\sin(kx - \omega t)$ at t: $kx - \omega t = 0$ when $x = \frac{\omega t}{k}$

$\sin(kx + \omega t)$ goes other way