**Announcements 2/23/10**

PS 5 will be posted later today. (With 5 hours of office hours yesterday, didn't get it finished.)

Reading for today: 23.2, 23.3 up to "Got It?" 23.1 (we are skipping p. 389 and example 23.3; we are doing much more with dielectrics than Wolfson does)

Reading for Thursday: 23.4 but not example 23.5; 24.1

Dr. Bennett Lorber, professor at Temple School of Medicine, will speak about careers in academic medicine Wednesday (tomorrow) 4:30 p.m. SC 199.

Feedback questionnaire from last week:

- Many helpful suggestions
- Common thread: pace in optics was good, pace in electricity so far has been fast
- Adjusting pace (material also gets easier)
- At end of class today, will ask you to write down a quick response
- Life science applications are not all in the book (or *any* introductory physics textbook); lecture notes online, if there is a topic for which you would find a more detailed handout useful, let me know
- Learning strategies detailed in syllabus
- I know this timeslot is difficult; scheduled at this time to minimize conflicts with biology & chemistry classes; feel free to bring a snack if you want!

## Key ideas from last time

Goal of this section:
Understand how energy is stored in charge arrangements

$q_1 \xrightarrow{r_{12}} q_2$

Electric potential energy of a pair of charged particles $U^E_{\text{charges}}$: energy required to bring them from at rest infinitely far apart to at rest in their final position.

Think of moving $q_2$ from infinitely far away to its final location.

An external force must do work to change the energy of the system. Particles are at rest, so energy is all potential:

$$\mathrm{W}_{\text{ext}} = U^E_{\text{charges}}$$

Change in electric potential energy = negative of work done by $E_1$ on $q_2$

$$W_{ext} = U^E_{\text{charges}} = -W^E = q_2 \underbrace{\Delta V_{\infty 2}}_{\text{potential diff produced by } q_1} = q_2 \overbrace{V_1(r_{12})}^{\text{potential produced by } q_1 \text{ at } r_{12} \text{ away}}$$

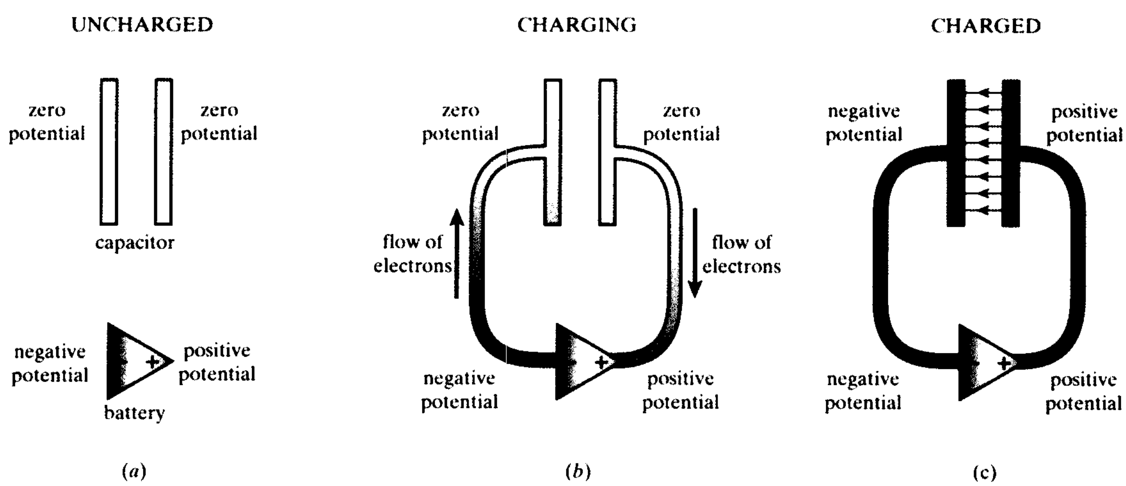
$$= q_2\left(\frac{kq_1}{r_{12}}\right)$$

$q_1 \xrightarrow{r_{12}} q_2$

Adding another charge causes an additional change in energy; use superposition to find the potential produced by both $q_1$ and $q_2$.

## Charging a capacitor

- Both plates ("electrodes") begin electrically neutral
- Both plates are at same potential, as there is no charge around and hence E = 0.
- Battery: internal chemical reaction gives one terminal a negative charge, other terminal a positive charge
- Conductors have free electrons that can move in response to an electric field — if nonzero $\Delta V_{AB}$, there must be an $\vec{E}$!
- Electrons will move to reduce electric potential energy, so that $\Delta U_{AB} = q\Delta V_{AB}$ is **negative** (like rolling downhill)




→ charge flows when capacitor plates are connected to battery terminals with conducting wires

Negatively charged electrons move to **higher** potential! (positively charged ions move to lower potential)

J.J.
2/23/2010

last time: electric potential energy of point charges
~~[illegible]~~ energy of a pair of ions: potential energy change ~~[illegible]~~ ~~[illegible]~~
on bringing together from $\infty$ far apart

idea of storing energy in an arrangement of charge —
how much energy does it to take to go from uncharged to charged?

Today:
1. Introduce capacitor, find energy stored in it
2. Effect of filling space between capacitor plates with polarizable material

Capacitor:
pair of conducting plates separated by a short distance that can be used to store energy by giving the plates opposite charges
(show them cap) (Eric's 1st fig)

Stores energy because it requires energy to pull charges apart

Charging: go to overhead file

How much charge do you have when reach $\Delta V_{batt}$?

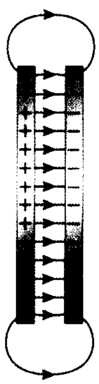
**How much energy can we store in the charged capacitor?**

Depends on:

- potential difference between plates $\Delta V_{cap}$ (equals $\Delta V_{batt}$)
- amount of charge on positively charged plate $+Q$ (Total charge of capacitor is zero!)

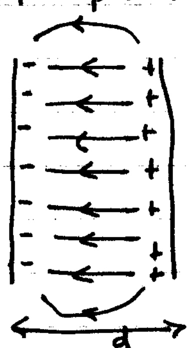
Find relationship between $Q$ and $\Delta V_{cap}$, then use that to find total energy

Capacitor electric field: like two very large sheets of charge with opposite sign



To find amount of charge for given $\Delta V_{cap}$, relate $\vec{E}$ to $\Delta V_{cap}$, then $Q$ to $\vec{E}$

$\vec{E}$ of capacitor: assume plates are large compared to separation



(curves at edges as you saw in lab)

Conventionally $\Delta V_{cap}$ = from ⊖ to ⊕

$$\Delta V_{cap} = \Delta V_{-+} = -\vec{E}\cdot\Delta\vec{r}_{-+}$$

$\Delta\vec{r}_{-+}$ points right

$\vec{E}$ points left

$$\Delta V_{cap} = -\vec{E}\circ\Delta\vec{r}_{-+} = -E\Delta r_{-+}\cos 180° = +E\Delta r_{-+}$$

Call distance between plates $d$ $= +Ed$

$\Rightarrow$ field strength $E = \dfrac{\Delta V_{cap}}{d}$

Previously we found field strength between opp. charged plates to be $E = 4\pi k\sigma = \dfrac{\sigma}{\epsilon_0}$ (found by adding fields of two sheets)

$$\Rightarrow \frac{\sigma}{\epsilon_0} = \frac{\Delta V_{cap}}{d}$$

and $\sigma = \dfrac{Q}{A}$ with $A$ = plate area

$$\Rightarrow \frac{Q}{A\epsilon_0} = \frac{\Delta V_{cap}}{d} \Longrightarrow Q = \underbrace{\left(\frac{A\epsilon_0}{d}\right)}_{} \Delta V_{cap}$$

capacitance: amt of charge that can be stored for a given $\Delta V_{cap}$

$$\boxed{\text{Define } C \equiv \frac{A\epsilon_0}{d} \quad \Rightarrow Q = C\Delta V_{cap}}$$

Depends only on plate shape & spacing!

Demo: change $C$ w/variable cap

$\boxed{\text{CT1}}$ Fix $Q$, increase $d$: which of $\vec{E}$, $\Delta V_{cap}$, $C$ change?

$\boxed{\text{CT2}}$ Requires work to pull plates apart

$\boxed{\text{CT3}}$ Energy

Thickness of cell membr 1st measured by measuring its capacitance!

The plates of a parallel plate capacitor are separated by a distance *d*. The plates are given charges $+Q$ and $-Q$ and then the charged capacitor is disconnected from the battery. If we pull the plates apart, increasing the distance between them, which of the following change?

1. The electric field between the plates
2. The potential difference between the plates $\Delta V_{cap}$
3. The capacitance of the capacitor
4. Both the electric field and the potential difference
5. Both the electric field and the capacitance
6. Both the potential difference and the capacitance
7. All three
8. Need more information

C changes b/c $C = \frac{A\epsilon_0}{d}$

E does <u>not</u> b/c $E = \frac{\sigma}{\epsilon_0}$ and Q constant

$\Delta V_{cap}$ must therefore change b/c $\Delta V_{cap} = Ed$

Consider a parallel plate capacitor whose plates are given charges $+Q$ and $-Q$ and then the charged capacitor is disconnected from the battery. To then increase the separation between the plates:

1. positive work must be done by a mechanical force to separate the plates
2. the plates will move apart if not fixed in place
3. neither (1) nor (2) is correct

You have to pull:

your force is in same direction as motion of plates

$\therefore W_{\text{your force}}$ is $+$

Consider a parallel plate capacitor whose plates are given charges $+Q$ and $-Q$ and then the charged capacitor is disconnected from the battery. If we then increase the separation between the plates, the electric potential energy of the arrangement of charge on the capacitor is

1. greater than
2. the same as
3. less than

before the plates were pulled apart.

Energy conservation:

Do work on capacitor by mech force

→ stored energy (potential) changes by same amount as work

$$W_{ext} = \Delta U^E$$

Must do positive work to pull plates apart

$$\Rightarrow \Delta U^E > 0$$

Energy increases.

How much energy to go from uncharged to charged?
Calculate it the same way we found the energy of three point charges: find $\Delta U$ for each charge as it moves from its initial location to its final loc'n

BUT: now instead of bringing charge from $\infty$ far away, we move charge from $\ominus$ plate to $\oplus$ plate

Start w/plates uncharged, move charge $dq$, then move next $dq$, and so on

$$\underbrace{dW^{batt}}_{\text{energy provided by battery}} = \underbrace{dU_{cap}}_{\text{increase in stored elec. p.e.}} = dq\,\Delta V_{cap} = dq\left(\frac{q}{C}\right)$$

when the cap is <u>partly</u> charged to $\pm q$, then $\Delta V_{cap} = q/C$

This depends on how much charge is on the capacitor!

To find total stored energy: add up all $dq$'s: integrate!

$$U_{cap} = \sum_{\text{all } dq\text{'s}} dq\left(\frac{q}{C}\right) = \int_{q=0}^{Q} dq\,\frac{q}{C} = \frac{1}{2}\frac{q^2}{C}\Bigg|_{q=0}^{Q} = \frac{Q^2}{2C}$$

* <u>Important</u> thing here is idea of building up stored energy by transferring bits of charge sequentially *

And can use $Q = C\Delta V_{cap}$ to write 3 diff ways:

$$U_{cap} = \frac{Q^2}{2C} = \frac{Q\Delta V}{2} = \frac{1}{2}C\Delta V_{cap}^2$$

This much energy is provided by the battery to the capacitor. (Starts as chemical bond energy.)

Sparking cap demo

This energy can be <u>quickly</u> released by connecting the plates with a wire — $e^-$ gain KE, flow through wire, KE can be used to drive a motor or power another device

The lasers in the NOVA laser fusion experiment deliver $10^{14}$ W of power (roughly 100 times the total power output of all the world's electrical power plants) when on, but the lasers are only on for $10^{-9}$ s at a time. The energy to power the lasers is supplied by a bank of capacitors with total capacitance of 0.26 F, and the efficiency with which the energy is converted to light energy is about 0.17%. How much charge is on the positive terminals of the capacitors when fully charged?

Power = rate of energy delivery = $\frac{\text{energy}}{\text{time}}$ $\quad W = \frac{J}{s}$

Need Q, given C and information that we can use to find total energy

$$U_{cap} = \frac{Q^2}{2C} \Rightarrow Q = \sqrt{2CU_{cap}}$$

Total energy in lasers: power × time laser is on

$$= (10^{14}\ W)(10^{-9}\ s) = 10^5\ J$$

Total energy in capacitors: $U_{laser} = 0.0017\, U_{cap}$

$$U_{cap} = \frac{10^5\ J}{0.0017} = 5.9 \times 10^7\ J$$

$$Q = \sqrt{2(0.26\ F)(5.9 \times 10^7\ J)} = 5.5 \times 10^3\ C$$

$$\Delta V_{cap} = \frac{Q}{C} = \frac{5.5 \times 10^3\ C}{0.26\ F} = 22 \times 10^3\ V$$

What is problem asking for? Charge on ⊕ terminals of caps

What are we given? ~~energy~~ power & duration of laser pulse and efficiency of capacitor to laser energy conversion

So, need to relate stored energy to charge:

$$U_{cap} = \frac{Q^2}{2C} \Rightarrow Q = \sqrt{2C\,U_{cap}}$$

Now need to determine energy in laser pulse, and from that can determine energy in capacitors:

$$U_{laser} = P_{laser}\,\Delta t_{laser}$$ from reln power ⇔ energy

and $$U_{laser} = 0.0017\,U_{capacitors}$$ from efficiency

So: $$U_{laser} = (10^{14}\,W)(10^{-9}\,s) = 10^5\,J$$

$$U_{cap} = \frac{U_{laser}}{0.0017} = 5.9\times10^{7}\,J$$

and finally $$Q = \sqrt{2(0.26F)(5.9\times10^{7}J)} = 5.5\,kC$$

Units ☑

☑ very large (as would be expected)

$(\Delta V_{cap} = Q/C = 22\,kV)$

Reasonable? Hard to say....

Where is this energy?
- Can think of it as in the locations of all the charges
- Another way that's convenient: stored in the electric field itself – field has energy

Rewrite stored energy in terms of E and volume:

$$U_{cap} = \frac{1}{2} C(\Delta V_{cap})^2 = \frac{1}{2} A \frac{\epsilon_0}{d} (Ed)^2 = \frac{1}{2} \epsilon_0 \underbrace{Ad}_{\text{volume}} E^2$$

so energy per volume is

$$u_E = \frac{U_{cap}}{\text{volume}} = \frac{1}{2} \epsilon_0 E^2$$

Every bit of space has energy per volume $\frac{1}{2} \epsilon_0 E^2$

$\boxed{\text{CT}}$ increasing size of plates @ fixed $\Delta V_{cap}$

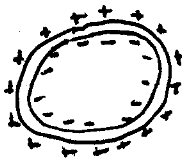
3 ways to explain:

(1) Increasing A increases C $\rightarrow$ U increases
(2) Increasing A increases volume occupied by $\vec{E}$
(3) Increasing A increases Q (required to keep $\vec{E}$ constant as $\vec{E} = \frac{\sigma}{\epsilon_0}$

Example: capacitors used in defibrillators, camera flashes, $\boxed{\text{NOVA laser fusion experiment}}$ – whenever you want to store a lot of energy to release quickly

Key biological example: cell membrane is like a capacitor in storing energy by layers of opposite charge

Main difference between cell membrane and this capacitor: charge layers are separated by polarizable material (lipid membrane)



We said that the electric field in the cell membrane is given by that of two charged sheets:

\+ + + + + + + + + + + + +

\- - - - - - - - - - - - -

$$E = 4\pi k \sigma_+ = \frac{\sigma_+}{\epsilon_0} \quad \text{with } \sigma_+ = \frac{Q}{A}$$

BUT actually the field is different because of the lipids between the charge layers — they polarize!

Fill in space with colored chalk
Show figure from Mazur (handout)

$\boxed{\text{CT}}$ How does total field change? <u>Less</u>: field due to polarized molecules points other way

$$\vec{E}_{total} = \vec{E}_{plates} + \vec{E}_{induced}$$

(Show fields w/ two colors of ~~chalk~~ pen
Draw in profile on handout)

We characterize how much the lipids polarize by defining the dielectric constant $\kappa$

If field ~~without dielectric would be~~ of the plates is $\vec{E}_{plates}$, then,
~~if the amount of charge on the plates is fixed,~~

$$\vec{E}_{total} = \frac{\vec{E}_{plates}}{\kappa} = \frac{4\pi k \sigma_+}{\kappa} = \frac{\sigma_+}{\epsilon_0 \kappa}$$

call density of charge on + layer $\sigma_+$

Note $k$ and $\kappa$ are different!!
~~Note~~ Stop using $k$ to avoid confusion w/ $\kappa$

$\kappa$ is a number greater than one – tells you how much ~~lipids~~ material polarizes – "dielectric constant"
for lipids in cell membrane, $\kappa \approx 8$

up to here

The charge densities on the outside and inside of a cell membrane are $\pm\sigma$. Accounting for the polarization of the material between the charge layers, the strength of the *total* electric field between the charge layers is

1. Greater than $4\pi k\sigma$. (field of plates alone)
2. Equal to $4\pi k\sigma$. $\leftarrow \frac{\sigma}{\epsilon_0}$
3. Less than $4\pi k\sigma$.
   $\uparrow \frac{\sigma}{\epsilon_0}$