Announcements 3/16/10

One more circuit problem on next homework; can turn in self-test 6 Friday afternoon this week.

Feedback questionnaire: extended deadline until Thurs for partial HW extra credit

Some results:
will add Friday afternoon help session,
single sheet with problems/questions
no formal system for collecting questions Tues/Weds:
- some prefer asking questions in person
- some are too busy

keep in mind: very important part of the learning process to articulate your questions (and ideas)!

Thursday evening: review of circular motion and cross products. SAs available for those who prefer to work on homework.

Reading:
Lab prep: 24.5 and 25.4 (plus a problem)
Today: 25.3 and 25.5
Thursday: 26.1-3

Exams: Please read carefully the summary comments online.

Key ideas from last time

Three principles of circuit analysis:
1. Potential differences around a loop of a circuit add up to zero (Conservation of energy; "loop rule")
2. Current is constant around a single-loop circuit; in a branching circuit, current into a node/junction = current out (charge doesn't pile up anywhere; "junction rule"/"node rule")
3. Ohm's Law:
voltage across resistor = current in resistor * resistance

$V_R = I_R R$

Usually: Think voltage (loop rule) first!

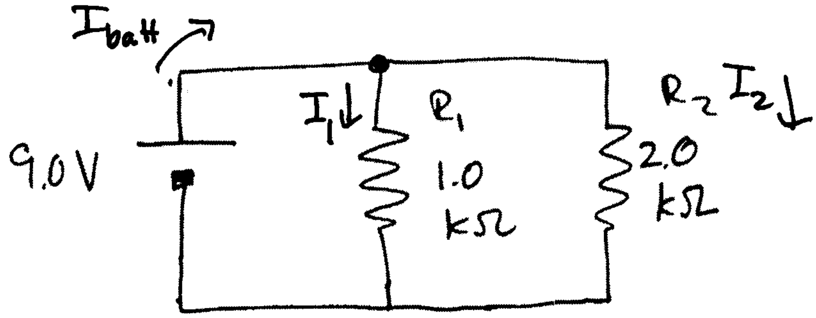
Strategy with multiple loops and one battery:

- Find equivalent resistance of circuit and from it find current in battery $I_{batt} = \mathcal{E}/R_{eq}$
- Then work your way back up to what you need to find using the three principles above
- Usually it is easiest to use the loop rule!

Series combinations: $R_{eq} = R_1 + R_2 + ...$

Parallel combinations: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ...$

Find the total current drawn from the battery in the circuit below.



loop rule left loop: $9.0\,V - I_1(1.0\,k\Omega) = 0$

$[\mathcal{E} - I_1 R_1 = 0]$

outer loop: $9.0\,V - I_2(2.0\,k\Omega) = 0$

$$I_1 = \frac{9.0\,V}{\underset{\uparrow\, 10^3\,\Omega}{1.0\,k\Omega}} = 9.0 \times 10^{-3}\,A$$

$$I_2 = 4.5 \times 10^{-3}\,A$$

$$I_{total} = I_1 + I_2 = 13.5 \times 10^{-3}\,A = 13.5\,mA$$

J.J.
3/16/10

Summary from last time
Usually think voltage first!
example of parallel circuit

$$\mathcal{E} - I_1 R_1 = 0 \quad \Rightarrow \quad I_1 = \frac{\mathcal{E}}{R_1} = \frac{9.0\,V}{1.0\,k\Omega} = \frac{9.0\,V}{1.0\times10^3\,\Omega} = 9.0\times10^{-3}\,A$$

$$\mathcal{E} - I_2 R_2 = 0$$

$$\Rightarrow \quad I_2 = \frac{\mathcal{E}}{R_2} = \frac{9.0\,V}{2.0\,k\Omega} = 4.5\times10^{-3}\,A$$

$$I_{batt} = I_1 + I_2 = 9.0\times10^{-3}\,A + 4.5\times10^{-3}\,A = 13.5\times10^{-3}\,A = 13.5\,mA$$

Today
1. More circuits with multiple loops and 1 battery
2. Short circuits: zero resistance path
3. Circuits with multiple batteries
4. ~~[illegible]~~ Circuits with capacitors
   - how current depends on time (RC "time constant")
   - discharging and charging

Multiloop circuit from last time:
- easiest way to find $V_{3k\Omega}$ is with loop rule (show loop on circuit
- to find $I_{3k\Omega}$ then use $V_{3k\Omega} = I_{3k\Omega}(3\,k\Omega)$ emphasize importance that the rest of the circuit is there)
- notes show how to do it dividing current

$\boxed{\text{CT}}$ compare $V_{3k\Omega}$ to $V_{2k\Omega}$

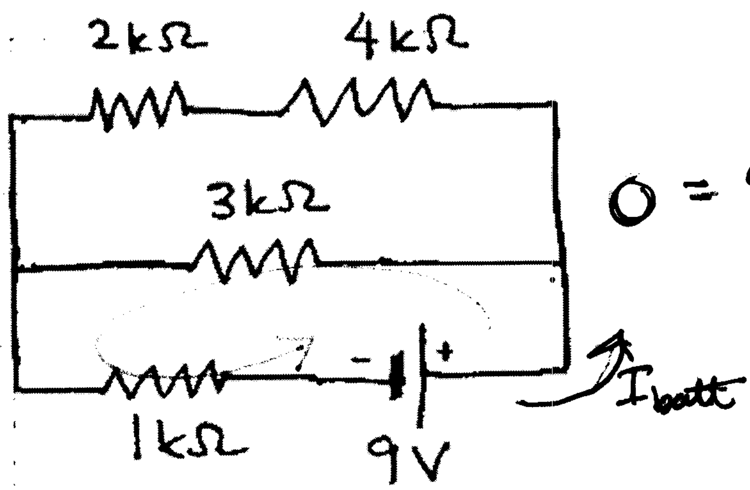
$V_{3k\Omega} = V_{2k\Omega} + V_{4k\Omega}$ so $V_{3k\Omega} > V_{2k\Omega}$

same current in $V_{2k\Omega}$ and $V_{4k\Omega}$ so ~~[illegible]~~ $V_{4k\Omega} = 2V_{2k\Omega}$

$$\Rightarrow \quad V_{3k\Omega} = V_{2k\Omega} + 2V_{2k\Omega} = 3\,V_{2k\Omega}$$

11:55

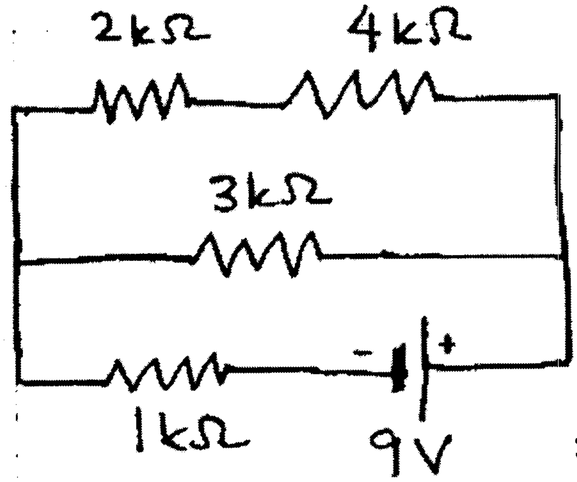
In the circuit below, find the voltage across the 3 kΩ resistor.



$$0 = 9V - V_{3k\Omega} - I_{batt}(1k\Omega)$$

1. Find R_{eq} of the branch with 2 kΩ and 4 kΩ.
2. Find R_{eq} of this branch combined with the 3 kΩ branch.
3. Find R_{eq} of the entire circuit, and use this to find the current in the battery.
4. Now return to the original circuit. **Use the loop rule with the lower loop to find $V_{3k\Omega}$.** [Or: Find the current in the branch with 3 kΩ by considering how the current splits between that branch and the branch with 2 kΩ and 4 kΩ, and use that current to find the potential difference across the resistor.]

In the circuit shown, how does the potential difference across the 3 kΩ resistor compare to the potential difference across the 2 kΩ resistor?



$V_{3k\Omega} = V_{2k\Omega} + V_{4k\Omega}$

Same current in 2kΩ & 4kΩ.

$V = IR$

$\Rightarrow V_{4k\Omega} = 2V_{2k\Omega}$

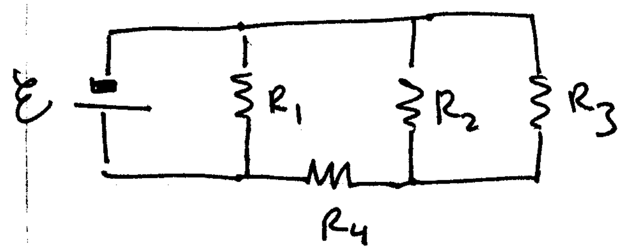
$\Rightarrow V_{3k\Omega} = V_{2k\Omega} + 2V_{2k\Omega}$

$V_{3k\Omega} = 3V_{2k\Omega}$

1. $V_{3k\Omega} > V_{2k\Omega}$
2. $V_{3k\Omega} = V_{2k\Omega}$
3. $V_{3k\Omega} < V_{2k\Omega}$
4. Do you have to do a long calculation?

If you finish early: find the ratio $V_{3k\Omega}/V_{2k\Omega}$

Question: combining resistors



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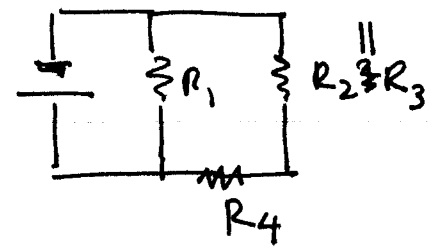

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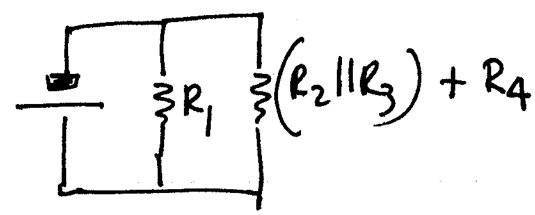


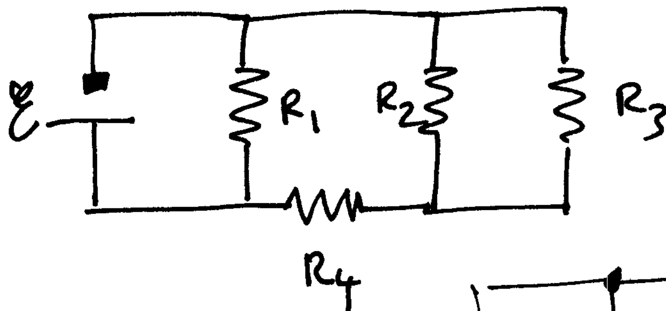
Current in R_4? Go to this ckt
Same as current in $(R_2 \| R_3) + R_4$ branch
Use

$$\mathcal{E} - I_{right} R_{right} = 0$$

You'll do this on HW (with values for R's and $\mathcal{E}$)
[Direction of current – on HW]

12:45

How do you combine resistors to find the equivalent resistance of the circuit below?



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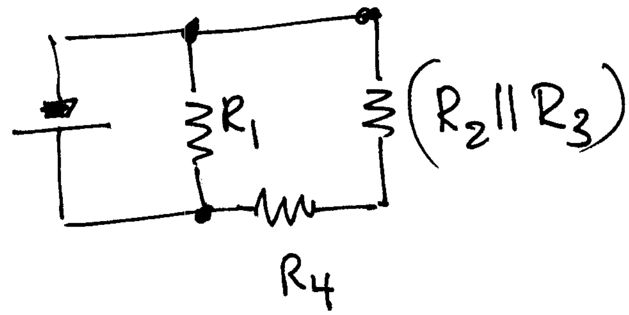

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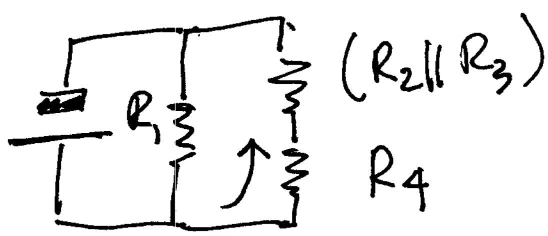

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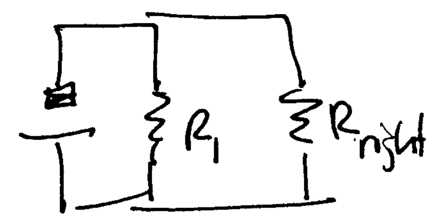

$R_{right} = R_4 + (R_2 || R_3)$

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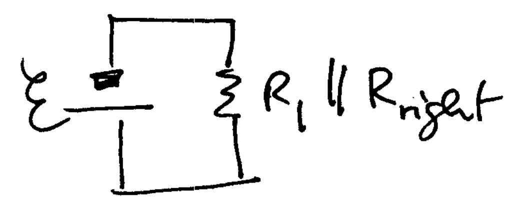


$\Rightarrow I_{batt} = \dfrac{\mathcal{E}}{R_{eq}}$

Find current in R_4

<u>Short circuit</u> : a zero-resistance conducting path

term used in two contexts

(1) As on HW: when terminals of a source (power supply / battery) are connected together just by a zero-resistance conductor

→ very large current not doing anything useful

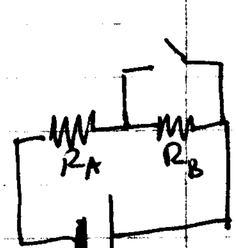
power = IV is also high → heats up

wasteful, potentially dangerous

(2) When $R = 0$ conducting path added in parallel to a resistor or other useful element of ckt.

DEMO

CT short ckt ⓘ bulb A gets brighter; bulb B goes out: all current through short



introduce switch symbol

3 ways to think about

(1) Voltage: short has $R = 0$ so <u>all</u> at constant potential; $V_{short} = 0$ (remember this is really ΔV_{short}!)

$\Rightarrow$ requires $V_B = 0$ also b/c in parallel

Then $P_B = I_B V_B = 0$

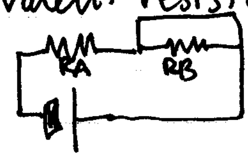
(2) Current: currents in ∥ branches satisfy

$I_B R_B = I_{short} R_{short} \Rightarrow \dfrac{I_B}{I_{short}} = \dfrac{R_{short}}{R_B}$

$R_{short} = 0 \Rightarrow I_B = 0$

Then $P_B = I_B^2 R_B = 0$

Equivalent <u>resistance</u> of ckt <u>decreases</u> — R_B just isn't there



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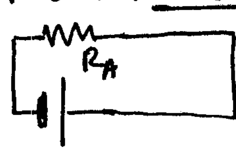
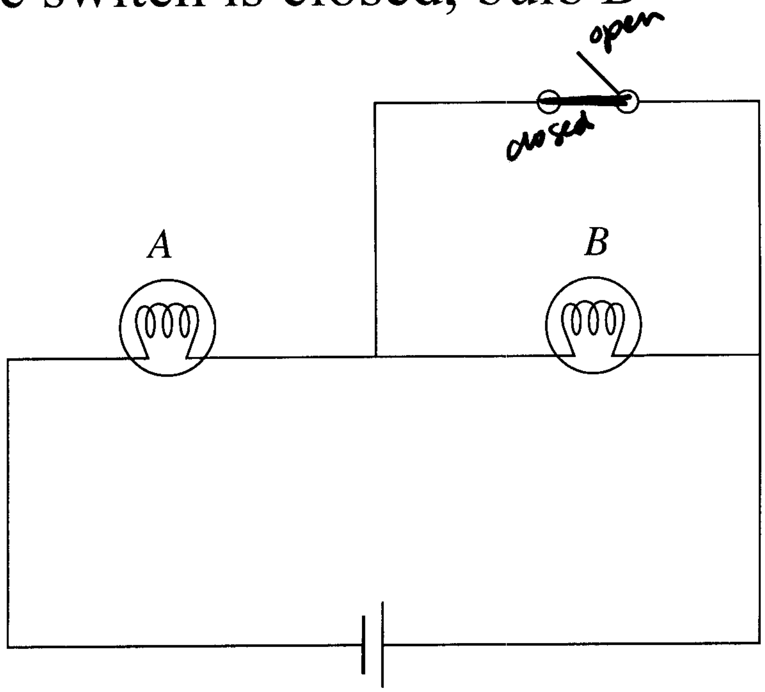


→ I_{batt} increases

When the switch is open as shown, the two identical light bulbs are equally bright. When the switch is closed, bulb *B*



why does it go out?
current follows path of least resistance

$V_{short} = 0$ b/c $R_{short} = 0$

$V_B = V_{short} = 0$ b/c parallel

Bulb brightness $= P_B = I_B V_B$
$= 0$

1. burns more brightly than before
2. remains unchanged
3. burns less brightly than before
4. goes out

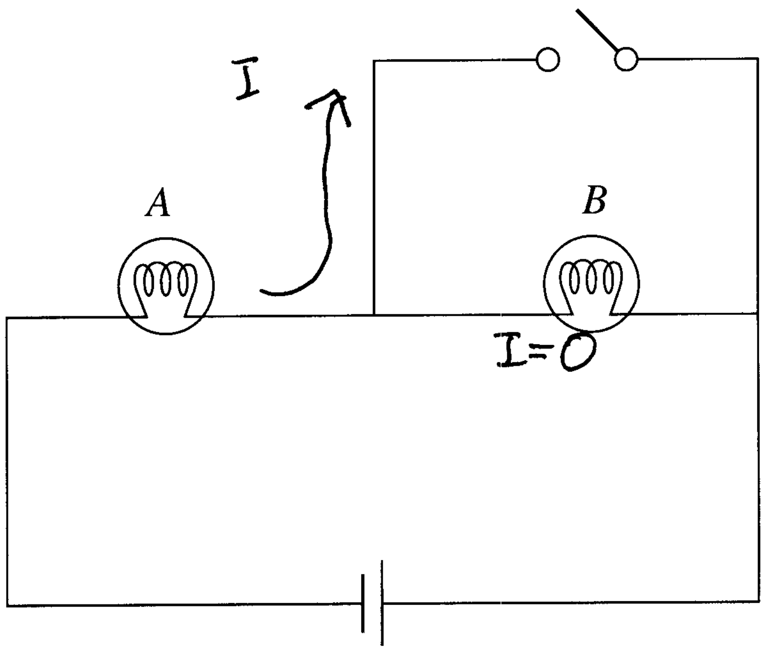
When the switch is open as shown, the two identical light bulbs are equally bright. When the switch is closed, bulb *A*

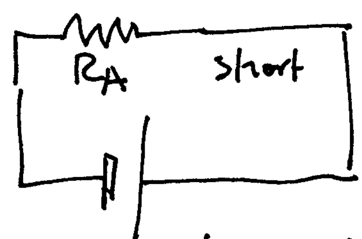
open switch:

$R_{eq} = R_A + R_B$

closed switch:

$R_{eq} = R_A + R_{short}$





$I_{batt} = \frac{\mathcal{E}}{R_{eq}} \uparrow$

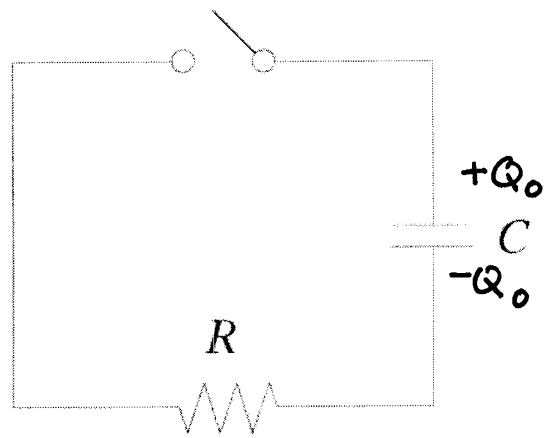
1\. ~~5.~~ burns more brightly than before

2\. ~~6.~~ remains unchanged

3\. ~~7.~~ burns less brightly than before

4\. ~~8.~~ goes out

A circuit consists of a light bulb with resistance R, a charged capacitor with charges $\pm Q_0$ on its plates, and a switch in series. The switch is initially open. After the switch is closed, the light bulb



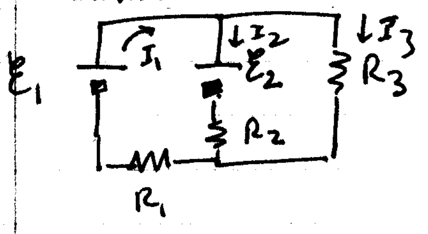
1. lights up with steady brightness.
2. lights up and grows gradually brighter. (I increases)
3. lights brightly at first, then gets dimmer and goes out.
4. does not light because there is no battery in the circuit.

(I decreases)

Circuits w/more than one battery

Draw on board first

Problem:



Find current I_2 in $\mathcal{E}_2$ and R_2, and identify its direction

$\mathcal{E}_1 = \mathcal{E}_2 = 9\,V$ (9.00 V)

$R_1 = R_2 = R_3 = 300\,\Omega$

Now we can't reduce this to an equivalent circuit of just 1 battery & 1 resistor — must instead write several eqs and solve.

Use loop rule and node rule

(2) How many equations do we need?

As many as there are unknown currents, resistances, and battery voltages (emfs) in the circuit.

- Know both batt $\mathcal{E}$
- Know all R's
- Don't know any currents

(1) How many distinct currents? 3 (add to figure)

How do we know which way to draw currents?

with just 1 battery, current always goes away from ⊕, toward ⊖ of battery

BUT w/multiple batteries we don't know!

$\boxed{\text{Make a guess — your guess will determine how to write the loop rule eqs.}}$

If you end up w/ a ⊕ current you guessed right

⊖ current means current goes the other way

Strategy:

1. Label currents.
2. Write down node rule for currents. ~~and~~ 3. Write as many loop rules as you need to get enough equations.
   Signs in loop rule are determined by directions of currents

For this problem we need 3 eqs b/c there are 3 unknown quantities

Node rule: $I_1 = I_2 + I_3$

Need 2 loop rules: choose loops that include I_2

Left loop: $\mathcal{E}_1 - \mathcal{E}_2 - I_2 R_2 - I_1 R_1 = 0$

battery emfs: add if cross battery from ⊖ terminal to ⊕ terminal
subtract if cross battery from ⊕ to ⊖
resistors: subtract if cross in same direction as current
add if cross opposite to current

(add when gaining energy
subtract when losing energy)

$\boxed{CT}$ Right loop $\mathcal{E}_2 - I_3 R_3 + I_2 R_2 = 0$

3. Goal is to find I_2: use these 3 eqs and eliminate I_1 and I_3 from node rule

Notice left loop has I_1 & I_2, right has I_2 & I_3

Solve left loop rule for I_1:

$$I_1 = \frac{\mathcal{E}_1 - \mathcal{E}_2 - I_2 R_2}{R_1} \quad \text{sub in values:} \quad = \frac{9V - 9V - (300\Omega) I_2}{300\Omega} = -I_2$$

Tells you either I_1 or I_2 is drawn in wrong direction

Solve right loop rule for I_3:

$$I_3 = \frac{\mathcal{E}_2 + I_2 R_2}{R_3} \quad \text{sub in values} \quad = \frac{9V + 300\Omega I_2}{300\Omega} = 3.0 \times 10^{-2} A + I_2$$

Substitute into node rule:

$$I_1 = I_2 + I_3$$

$$\Rightarrow \quad -I_2 = I_2 + 3.0\times10^{-2}\,A + I_2$$

Solve for I_2:

$$-3I_2 = 3.0\times10^{-2}\,A = 30\,mA$$

$$I_2 = -1.0\times10^{-2}\,A = -10\,mA$$

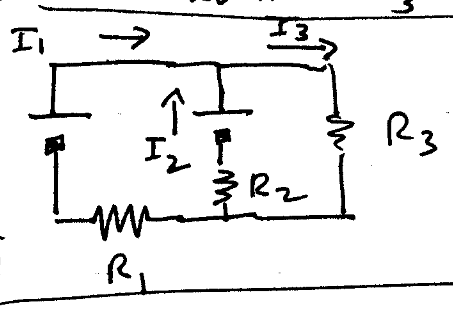
$\Rightarrow \; I_1 = -I_2 = +10\,mA$ ~~and $I_3 = -30$ mA & $I_3 = 20$ mA~~

Tells us I_1 is drawn in the correct direction, but I_2 is not

Finally $\quad I_1 = I_2 + I_3$

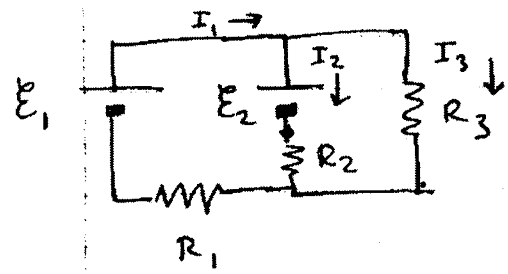
$$\Rightarrow \quad 10\,mA = -10\,mA + I_3$$

$$20\,mA = I_3$$



has currents drawn in correct direction so with this, $I_2 = +10\,mA$

For the circuit below, write the expression for the sum of the voltages around the right loop (loop that includes $\mathcal{E}_2$, R_2 and R_3).



CW around right loop crossing R_2 opposite I_2 $\Rightarrow$ add I_2R_2 (charge gains energy moving this way)

1. $\mathcal{E}_2 - I_3R_3 - I_2R_2 = 0$
2. $\mathcal{E}_2 + I_3R_3 + I_2R_2 = 0$
3. $\mathcal{E}_2 - I_3R_3 + I_2R_2 = 0$
4. $\mathcal{E}_2 + I_3R_3 - I_2R_2 = 0$
5. Other

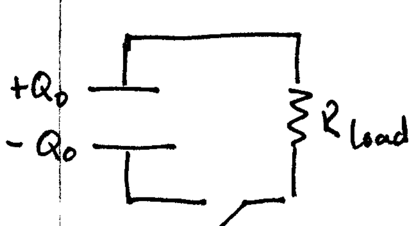
12:25 Discharging capacitors

We said capacitors were a way to store energy and then release it in a quick burst of current

How quickly can we release it?

Usually we want to put the current through something that has resistance (light bulb, defibrillator)

Draw circuit representing situation: charged capacitor, initially no way for it to discharge



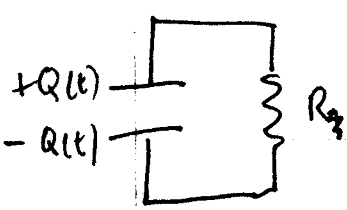
close switch $\rightarrow$ current can flow from ⊕ to ⊖ plate through resistor, until no net charge on either plate

$\boxed{\text{How does current depend on time?}}$

$\boxed{\text{CT}}$ If R_{load} is a light bulb: lights up brightly at first, then gets dimmer & goes out

How understand?

Write loop rule for ckt after switch closed at $t=0$



$$V_{cap} - IR = 0$$

$$\frac{Q(t)}{C} - I(t)R = 0 \Rightarrow \frac{Q(t)}{C} = I(t)R \Rightarrow I(t) = \frac{Q(t)}{RC}$$

as Q decreases, I decreases too!

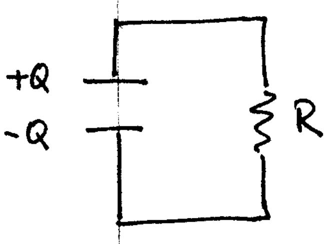
largest I is at $t=0$ when close switch:

$$I_0 = \frac{Q_0}{RC}$$

as current decreases, power in bulb $\downarrow$

Demo: capacitor hoist - motor turns more slowly as time goes on

How does current vary with time quantitatively?



Choose $t=0$ as instant when switch is closed and current begins to flow

Write loop rule: $\frac{Q(t)}{C} - I(t)R = 0$

$$\Rightarrow \quad Q(t) = I(t)\,RC$$

To find $I(t)$, rewrite making use of relationship between $I(t)$ and $Q(t)$

- current comes from <u>loss</u> of charge from ⊕ plate (amt of charge passing any point in time dt is $-dQ$)

$$\Rightarrow \quad I(t) = -\frac{dQ(t)}{dt}$$

got to here

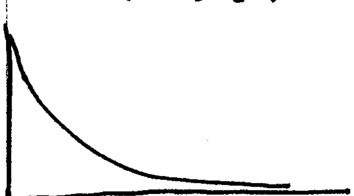
Differentiate loop rule: $\frac{dQ}{dt} = \frac{dI}{dt} RC$

~~using this~~

Substitute for $\frac{dQ}{dt}$: $-I(t) = \frac{dI(t)}{dt} RC$

Integration

$$\Rightarrow \quad I(t) = I_0 e^{-t/RC} \qquad I_0 = \text{current at } t=0$$



How graph it?

starts at maximum I_0

decrease <u>slows down</u> as it progresses

why? { less charge → smaller V_{cap} → smaller current
as current decreases, lose charge more gradually → current decreases more slowly also

Rule of thumb: $e^{-1} \approx 0.37$ so at $t = RC$, $I(t=RC) = \frac{I_0}{e} \approx 0.37\, I_0$