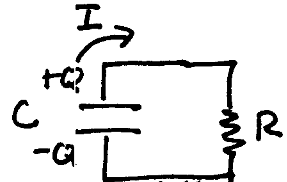
3/23/10

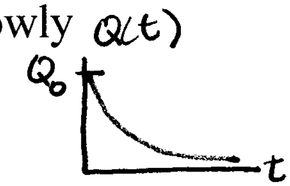
Key ideas from last time

Discharging capacitor:



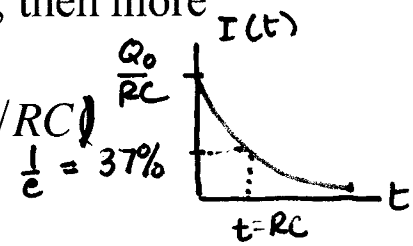
Charge on capacitor decreases rapidly at first, then more slowly

$$Q(t) = Q_0 e^{-t/RC} \quad \text{(find voltage from } V_{cap} = Q/C\text{)}$$



Current is greatest initially; decreases rapidly at first, then more slowly

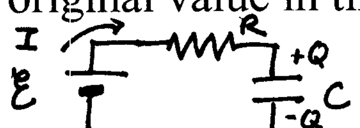
$$I(t) = I_0 e^{-t/RC} \quad \text{(initial current: } I_0 = V_0/R = Q_0/RC\text{)}$$



Rate of decrease determined by RC

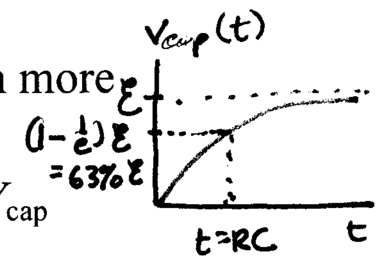
Decreases to 37% ($1/e$) of original value in time = RC

Charging capacitor:



Voltage across capacitor *increases* rapidly at first, then more slowly, toward battery voltage

$$V_{cap}(t) = \mathcal{E}\left(1 - e^{-t/RC}\right) \quad \text{find charge from } Q = CV_{cap}$$

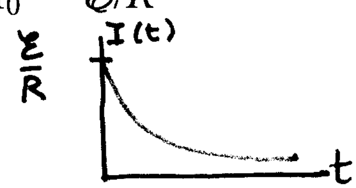


Rate of increase determined by RC also

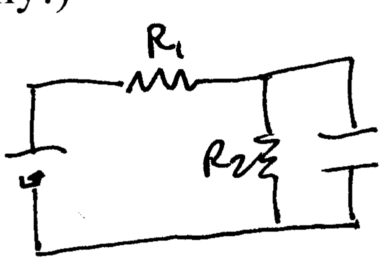
Increases to (1 - $1/e$) = 63% of original value in time = RC

Current:

As with discharging, $I(t) = I_0 e^{-t/RC}$ with initial current $I_0 = \mathcal{E}/R$



(In charging circuit, $V_R = IR = \mathcal{E} - V_{cap}$ which decreases steadily!)



Electrical signals in nerves

Consider just two cases:

(1) How "resting potential" = charged cell membrane is established — nerve is not signaling

(2) ~~a couple of features of~~ How membrane R & C affect speed of signals travel in nerves

2 types of ion channels (1) Always open

These involve different circuits b/c there exist (2) gated ion channels that can open in response to stimulus to allow current to flow across membrane. Changes in "membrane potential" (ie ΔV across membrane) trigger these opening.

---

(1) Resting nerve cell: no current flowing along it

What determines "resting potential" = ΔV across membrane

Show Cotterill: axon is long cyl cell, ⊕ outside ⊖ inside at rest ~~no current traveling along so~~ focus on R, C, $\mathcal{E}$ that cross membrane

(note 12.1 is missing batteries — not uncommon in neuro books to show just part of ckt)

Think of axon as made up of segments each w/ its own ~~[illegible]~~ equiv ckt — Fig 12.5

Focus on stuff that goes inside to outside:

- 3 batteries: each type of ion (K, Na, Cl) ~~[illegible]~~ is pumped by an ion pump to establish a certain concentration inside & outside cell
  - difference in interior & exterior conc: "concentr gradient"
  - → pot diff "Nernst potential"
  - each ~~[illegible]~~ corresponds to its own battery
- 3 resistors: each ion type also leaks across membrane through one type of ion channels that are open all the time
- capacitor: membrane capacitance


means variable resistance

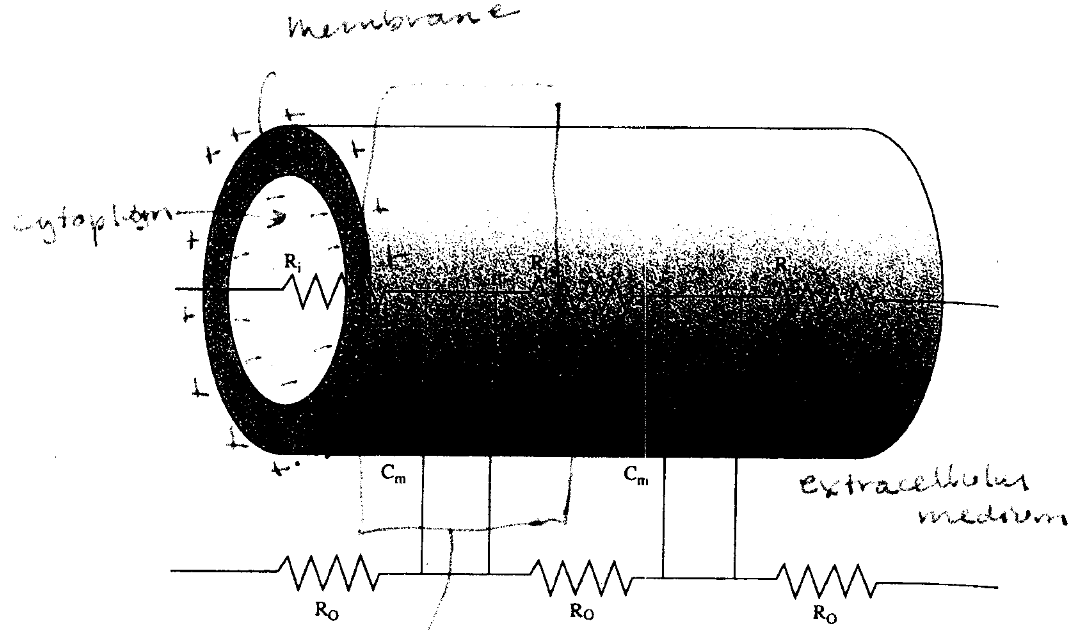


Figure 12.1 Idealized representation of a nerve process (axon or dendrite) showing the various associated resistances and capacitances

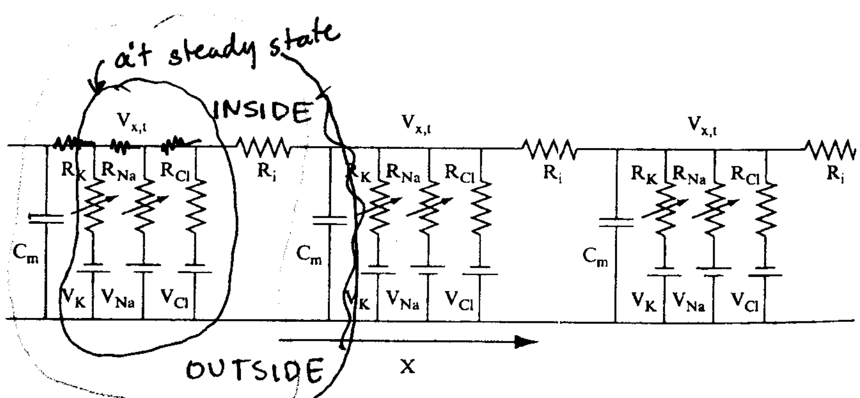


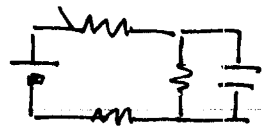
Figure 12.5 The ~~equivalent~~ circuit for a nerve cell process when the extracellular resistance can be ignored, but when the voltage dependences of the various ionic conductivities must be allowed for

Focus on what crosses from inside to outside → $\Delta V_{out\ in}$

3 ~~batteries~~: one for each type of ion
  ion pumps move each type across membrane → particular concentrations in & out
    diffs in concentration ("concentration gradients")
      → $\mathcal{E}$'s of batteries

3 resistors: (variable resistor symbol) means variable resistance
{ 2 types of ion channels allow ions to flow across
{ - open all time
  - "gated" channels that open only for particular $\Delta V_{out\ in}$

C_m: charges up so its voltage matches $\Delta V_{out\ in}$ – reaches steady state

Last time: 

When switch closed for a long time, cap fully charged —
no current in its branch

→ membrane potential $\Delta V_{out\ in}$ determined by this ckt!
(circle part w/ resistors & batts) (Your HW problem)
resistances of ion channels & concentration gradients
set up by ion pumps
[Mathematical biologists model these ckts]

C_m is charged to whatever $\Delta V_{out\ in}$ is set by batts & resistances

Note: often hear term "conductance" used — $G = 1/R$
(used b/c as $G \uparrow$, current $\uparrow$ — more current for same ΔV
if G is large) $I = G\Delta V$

(2) Travel of action potential (nerve signal)
Stimulus of some kind (pain, hot/cold, idea) triggers (2nd type of) gated ion channel to open — dramatically reduces resistance, ~~but only in small region where stimulus arrives~~

Na^+ channel is the one to open → Na^+ goes inside, reducing $\ominus$
→ brings $\Delta V_{out\ in}$ to $\oplus$ value

BUT this only happens in the small area where stimulus arrives
How does the signal spread down the nerve? Show Fig 8-6
As $\Delta V_{out\ in}$ changes in one region, change triggers
adjacent gated channels to open → signal spreads

What determines speed of travel?
Think about how current from the initial stimulus spreads

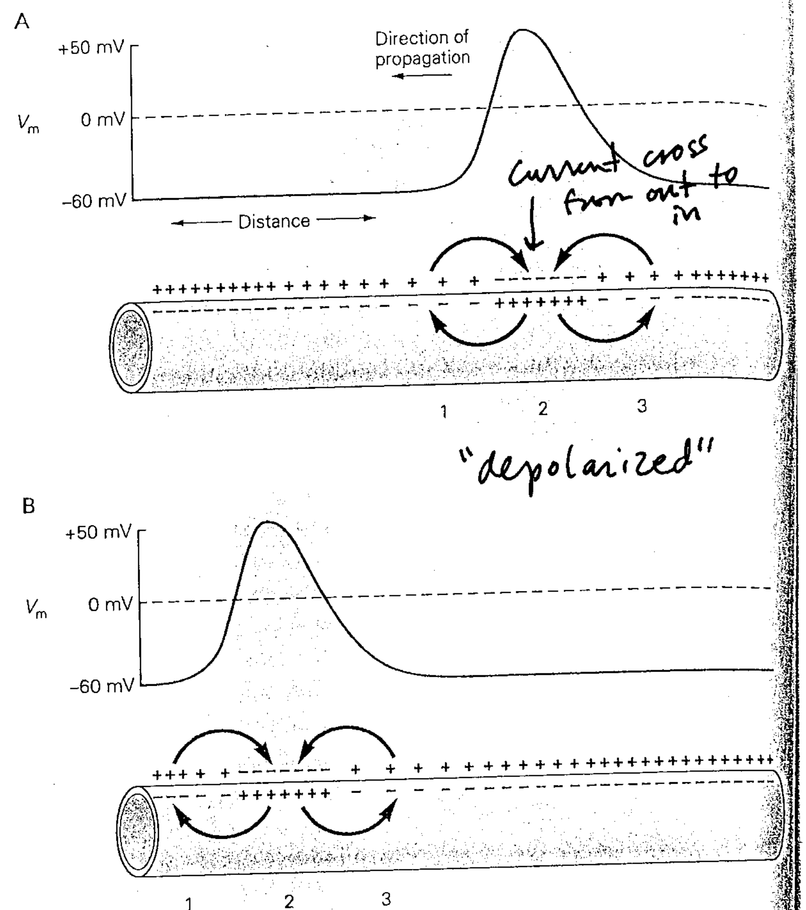


Figure 8-6 **Passive conduction of depolarization along the axon contributes to propagation of the action potential.**

A. The waveform of an action potential propagating from right to left. The difference in potential along the length of the axon creates a local-circuit current flow that causes the depolarization to spread passively from the active region (2) to the inactive region *ahead* of the action potential (1), as well as to the area *behind* the action potential (3). However, because there is also an increase in g_K in the wake of the action potential (see Chapter 9), the buildup of positive charge along the inner side of the membrane in area **3** is more than balanced by the local efflux of K^+, allowing this region of membrane to repolarize.

B. A short time later the voltage waveform and the current distributions have shifted down the axon and the process is repeated.

Kandel, Principles of Neuroscience

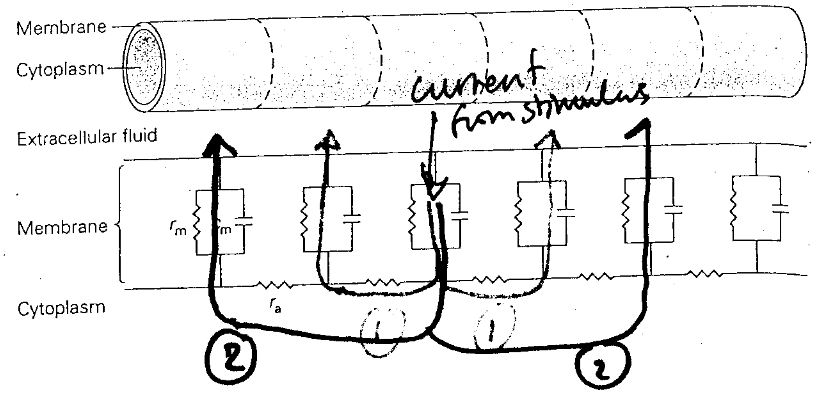


Figure 8-4 A neuronal process can be represented by an electrical equivalent circuit. The process is divided into unit lengths. Each unit length of the process is a circuit with its own membrane resistance (r_m) and capacitance (c_m). All the circuits are connected by resistors (r_a), which represent the axial resistance of segments of cytoplasm, and a short circuit, which represents the extracellular fluid.

$R_1 < R_2$

If cytoplasmic resistance r_a << membrane r_m

then R_1 is not a lot less than R_2

→ nearby paths are very similar in resistance → signal spreads a long way

Kandel, Principles of Neuroscience

Stimulus comes across membrane

→ current across membrane at one location

More current along cell

→ more ⊕ charge on inside

→ extends region of depol

travels farther for smaller Ra

How fast?

r_a

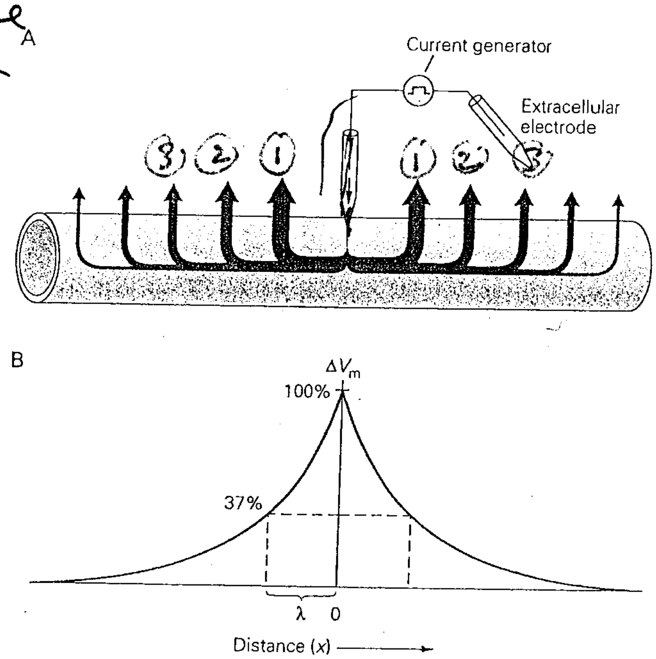


Figure 8-5 The voltage response in a passive neuronal process decays with distance due to electronic conduction. Current injected into a neuronal process by a microelectrode follows the path of least resistance to the return electrode in the extracellular fluid (A). The thickness of the arrows represents membrane current density at any point along the process. Under these conditions the change in V_m decays exponentially with distance from the site of current injection (B). The distance at which ΔV_m has decayed to 37% of its value at the point of current injection defines the length constant, λ.

As current comes into cell must go somewhere
To figure out both how far & how fast it spreads:
consider both resistance <u>inside</u> (r_a) & resistance across membrane (r_m)

Dark arrows show paths followed by current
More current goes nearby (1) than further (2) b/c
farther paths have more resistance
current changes capacitor C_m charge → spreads change in membrane potential

BUT if $r_m \gg r_a$ most current goes along cell, less across → spreads further (less difference btw (1) & (2))

Also small r_a → spreads faster
RC time for charging/discharging C_m is $r_a C_m$ b/c r_a is in series w/ C_m
as last time: ~~for~~ fixed C_m,
~~want big C_m to store lots of charge~~
→ small r_a lets it discharge fast

What are the physiological consequences?

(1) Large diameter nerves → faster signals b/c r_a smaller
$(R = \frac{\rho L}{A})$ A = xs area (C increases like diameter
R decreases like diam^2)

(2) Myelination of nerves → faster b/c reduces C
thick layer of myelin — larger distance in to out
$$C = \frac{K A \epsilon_0}{d}$$ A = area of cell membrane
→ C decreases

Today's goals: (0) Introduce magnetic field $\vec{B}$

(1) Magnetic force on moving charged particles: $\vec{F} = q\vec{v} \times \vec{B}$

(2) Magnetic force on a current in terms of current: $\vec{F} = I\vec{L} \times \vec{B}$

Example: mass spectrometer

Example: magnetic levitation

Magnetism

Magnetic materials (bar magnets) have been known for a long time!

Originally (in 18th century and before): magnetism was thought to be a completely different phenomenon from electricity because magnets do not exert forces on stationary charged objects

(Magnet does not attract or repel a charged capacitor, for example)

Magnets known to ~~[illegible]~~ attract/repel each other and to respond to the Earth's magnetic field —

compass = sliver of magnet free to turn on a mount
turns to point its "north" end toward Earth's magnetic North Pole
(sliver is parallel to magnetic field)
if there is a magnet nearby, it ~~brings~~ turns to its "north" end nearer the magnet's "south" end

First clue that magnetism and electricity are related:
(1820, Danish physicist Oersted) compass also turns when near a current-carrying wire
- if no current, compass follows Earth's mag field
- switching on current → compass turns!

DEMO circle of compass needles around wire

In addition: a wire made of copper or aluminum with no current feels no force from a magnet
BUT switching on a current in the wire → force!

DEMO

Big picture goals for next two weeks:
(1) Explain why bar magnets are magnetic - where magnetism in ~~materials~~ atoms comes from
(2) Explain how to make "electromagnets" with current carrying wires — powerful magnets for MRI all made w/ currents!

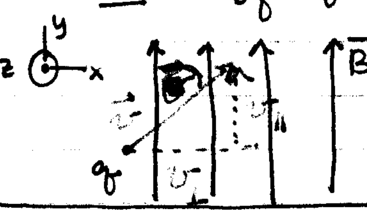
<u>Primary life science applications</u>:
(1) Basis of NMR and MRI: how atoms respond to magnetic fields
(2) Magnetic sensing in animals (how do animals navigate?)

Begin with simplest phenomenon to understand: magnetic forces on moving charged particles
Take magnetic field as a given for now - we'll explain it on Thursday, and introduce how magnets interact w/each other

Define magnetic field $\vec{B}$ in same manner as electric field $\vec{E}$:
- produced by a magnet fills space around it
- $\vec{B}$ tells us how strong a force is exerted on <u>other</u> magnets, moving charges, or currents
- has both a strength and a direction at every point in space

Key is that magnetic field exerts force on charged objects <u>only if they are moving</u>!

Force on charged particle w/charge q: <u>NOT</u> $\vec{F}_{Bq} = q\vec{B}$!
Instead: $\vec{F}_{Bq} = q\vec{v} \times \vec{B}$

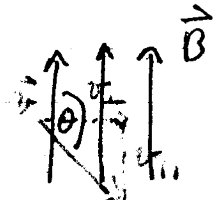


Cross product: $\vec{v} \times \vec{B} = vB\sin\theta\ \hat{k}$
if $\vec{v}, \vec{B}$ in xy plane $= v_{\perp} B\ \hat{k}$

$\boxed{\vec{F} \text{ always } \perp \text{ to plane containing } \vec{v}, \vec{B}}$

Introduce ⊙ out of board (arrow point)
⊗ into board (arrow feathers)

RH rule for direction of force: fingers of RH on $\vec{v}$
curl to $\vec{B}$
thumb points along $\vec{v} \times \vec{B}$



$\boxed{\text{If } q \text{ negative, } \vec{F} \text{ opposite } \vec{v}\times\vec{B}\,!}$

$\boxed{CT\ 1}$ q moving in $\vec{B}$: $\vec{F}$ is out of page

$\boxed{CT\ 2}$ deflection of beam $\Rightarrow$ $\vec{B}$ points ~~out~~ into of page

Recall work = change in energy = $\int \vec{F} \cdot d\vec{r}$

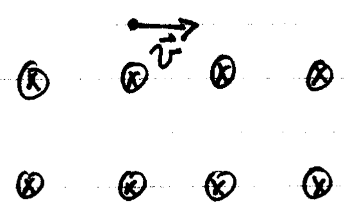
Because $\vec{F}_B \perp \vec{v}$, <u>no work is done</u> and thus the energy of the particle does not change

direction of motion changes but <u>not</u> speed!

~~$\Rightarrow$ uniform circular motion if $\vec{v} \perp \vec{B}$~~

just like circular motion due to a centripetal force

<u>If $\vec{v} \perp \vec{B}$:</u>



$\vec{B}$ into page

$\vec{F}$ always $\perp$ $\vec{B}$ and $\vec{v}$ $\Rightarrow$ turns particle in circular orbit

Equate strength of magnetic force

$F = qvB\sin\theta = qvB$ b/c $\theta = 90°$

to magnitude of centripetal force

$F_c = \dfrac{mv^2}{r}$ with v = speed, r = radius of circle

Do this for lab warmup

$\Rightarrow$ radius $r = \dfrac{mv}{qB}$

large $v \rightarrow$ large circle

large $B \rightarrow$ small circle

Key: depends on q/m (charge to mass ratio of particle), speed, field strength

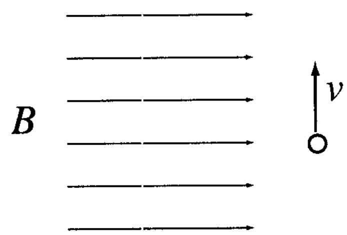
(save for later)

How long does it take to go in circle?

Period T of orbit $= \dfrac{\text{circumference}}{\text{speed}} = \dfrac{2\pi r}{v} = \dfrac{2\pi\left(\frac{mv}{qB}\right)}{v} = \dfrac{2\pi m}{qB}$

Frequency $f = \dfrac{1}{T}$

A negative particle moves upward along the trajectory shown. A magnetic field points toward the right.

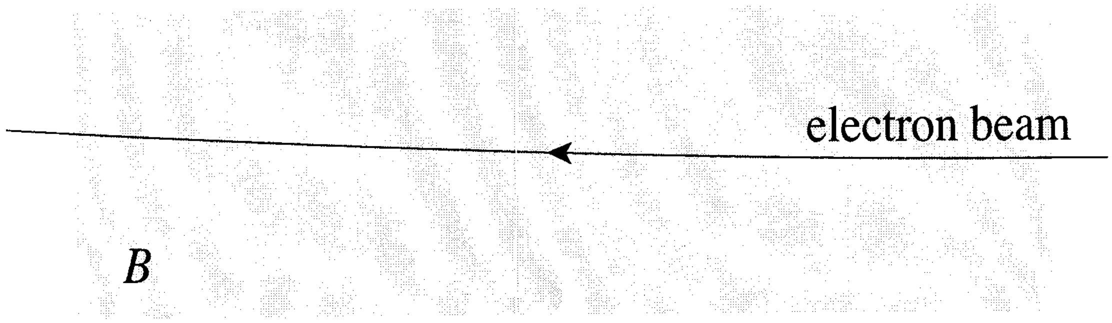


In which direction is the magnetic force on the particle?

$\vec{v} \times \vec{B}$ into page
q is negative
$\rightarrow \vec{F}$ out of page
$= q\vec{v} \times \vec{B}$
(opp. $\vec{v} \times \vec{B}$)

1. up
2. down
3. into the plane of the drawing
4. out of the plane of the drawing
5. left
6. right

A beam of electrons enters a region with a magnetic field as shown below.



If the beam is deflected upward, the magnetic field must be oriented

$-e\vec{v}\times\vec{B}$ up
so $\vec{v}\times\vec{B}$ down

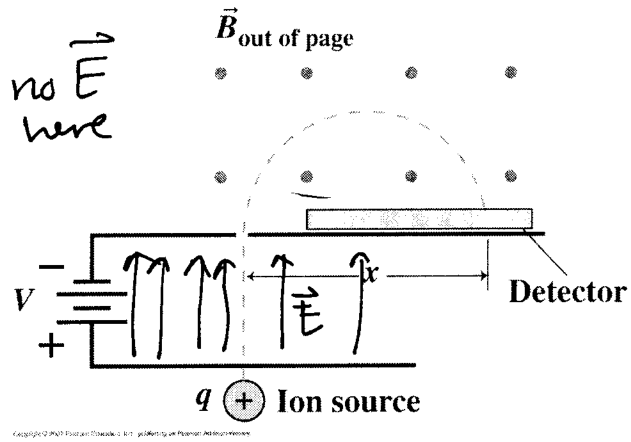
1. downward
2. up
3. into the plane of the drawing
4. out of the plane of the drawing
5. to the left
6. to the right
7. none of the above - it is at an angle
8. need more information to determine

A mass spectrometer is used to identify the charge-to-mass ratio of charged particles in the apparatus shown. The accelerating voltage is 10 kV. Protons used to calibrate the instrument land on the detector 10 cm from the entrance slit. What is the magnetic field strength?

Detector records where ions land

Mass spec uses distance traveled to determine q/m ratio of particles

no $\vec{E}$ here



Principle:

accel. voltage is known $\rightarrow$ gives ions speed v that just depends on charge and V

radius of circ. orbit is measured, related to q, m, v, B

Mass spectrometer

Use this principle to identify chemical species
- ionize molecules by placing in strong $\vec{E} \Rightarrow$ break up into charged fragments (always the same way)
- place fragments in system (show figure)
- particles follow half-circle path, then hit detector

Problem asks:
(1) Find $\vec{B}$ given that protons hit detector 10 cm from entrance slit
(2) What do we know about unknown ion?

Principle: protons/ions are accelerated from rest to some speed v by V_{accel}
then enter region of $\vec{B}$ and follow half-circle path (no $\vec{E}$ there)

Goal: (1) Find $\vec{B}$

Acceleration step: particles lose ΔU^E to gain ΔK

$$\Rightarrow \Delta K + \Delta U = 0$$

$$\frac{1}{2}mv^2 - q\Delta V_{accel} = 0 \qquad \text{(moving + to − so } \underline{\text{lose}} \text{ energy)}$$

Find v $$\Rightarrow v = \sqrt{\frac{2qV_{accel}}{m}}$$

Enter $\vec{B}$ region w/this speed: determines radius of trajectory
From before

$$r = \frac{mv}{qB} \qquad \text{Looking at figure, } x = 2r$$

Subst for v:

$$x = 2r = \frac{2m}{qB}\sqrt{\frac{2qV_{accel}}{m}} = \frac{2}{B}\sqrt{\frac{2mV_{accel}}{q}}$$

We are given $x = 0.10\ m$ $\quad m_{proton} = 1.67 \times 10^{-27}\ kg$

$V_{accel} = 10 \times 10^3\ V$ $\quad q_{proton} = +e = 1.6 \times 10^{-19}\ C$

Want B so solve above for B.

Units in MKS will give B in Tesla: $1\,T = 1\,\frac{N\cdot s}{C\cdot m}$

$$\Rightarrow B = \frac{2}{x}\sqrt{\frac{2mV_{accel}}{q}} = \frac{2}{(0.10\,m)}\sqrt{\frac{2(1.67\times10^{-27}\,kg)(10\times10^{3}\,V)}{(1.6\times10^{-19}\,C)}}$$

$$\boxed{B = 0.20\,T}$$

(0.204 but only 2 sig fig)

$$\frac{1}{m}\sqrt{\frac{kg\,V}{C}}$$

$$1\,V = 1\,J/C = 1\,N\cdot m/C = 1\,\frac{kg\,m^2}{C\,s^2}$$

$$\Rightarrow \frac{1}{m}\sqrt{\frac{kg\;kg\cdot m^2/Cs^2}{C}} = \sqrt{\frac{kg^2\,\cancel{m^2}}{\cancel{m^2}\,C^2 s^2}}$$

$$= \frac{kg}{C\,s} = \frac{(Ns^2/m)}{C\,s} = \frac{N\cdot s}{C\cdot m}$$

Unknown ion lands twice as far away:

$$\frac{x_{ion}}{x_{proton}} = 2 = \frac{\frac{\cancel{2}}{\cancel{B}}\sqrt{\frac{\cancel{2}m_{ion}\cancel{V_{accel}}}{q_{ion}}}}{\frac{\cancel{2}}{\cancel{B}}\sqrt{\frac{\cancel{2}m_{proton}\cancel{V_{accel}}}{q_{proton}}}} = \frac{\sqrt{\frac{m_{ion}}{q_{ion}}}}{\sqrt{\frac{m_{proton}}{q_{proton}}}}$$

~~Find charge to mass ratio~~

$$\Rightarrow 2\sqrt{\frac{m_{proton}}{q_{proton}}} = \sqrt{\frac{m_{ion}}{q_{ion}}}$$

Square

$$4\,\frac{m_p}{q_p} = \frac{m_{ion}}{q_{ion}}$$

so ion has $4\times\ m/q$ or $\frac{1}{4}\ q/m$ of proton

Measure charge separately by electrical means $\rightarrow$ determine mass of fragments

Announcements 3/18/10 (go over at end)

Friday afternoon office hours this week and next: 1:30 – 3 (no junior lab)

Reading:
Today: 26.3-4 (skip Hall effect)
Thursday: no new reading before class (we'll cover 26.5 quite differently than Wolfson)

PS 8: Preliminary version posted, finalize this afternoon.
Self-tests no longer need to be turned in (have achieved goal of familiarizing you with grading/expectations). Feel free to ask me for a grade!

Next exam is Thursday April 8; I will answer questions Tuesday night April 6, SAs available Wednesday night April 7

for Thurs

Feedback:
General purpose: Feedback allows me to hear from everyone in the class and to adjust a few aspects of how I am teaching the class to best match what I hear. With such a large class, there's a broad range!
Most important element: what *you* do