Announcements 3/18/10

Friday afternoon problem/question session: 3:30 – 5 at my office

Tonight's problem session:
7:30-8 review of circular motion and cross products, followed by homework.
SAs available for those who prefer to work on homework.

Reading:
Today: 26.1-2 (though probably we won't get to it)
Tuesday: 26.3-4

J.J. 3/18/2010

Last time: saw that as a capacitor discharges, at first it discharges rapidly (high current), and then as it continues, it slows down (less current).

Why? V_{cap} drives the current; as $V_{cap} = \frac{Q(t)}{C}$ decreases, so does the amount of current and thus how fast the cap discharges

Loop rule:

$$V_{cap}^{(t)} - I^{(t)}R = 0 \Rightarrow V_{cap}^{(t)} = I^{(t)}R \Rightarrow I(t) = \frac{V_{cap}(t)}{R}$$

Use $V_{cap}(t) = \frac{Q(t)}{C} \rightarrow$ $\cancel{Q(t) = I(t)R}$ $I(t) = \frac{Q(t)}{RC}$

~~So greatest current happens when charge is greatest~~.

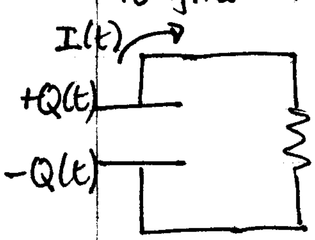
Define $t=0$ as the time when first close switch;

at $t=0$, $Q(t=0) = Q_0$

$$\Rightarrow I(t=0) = \frac{V_{cap}(t=0)}{R} = \frac{Q_0/C}{R} = \frac{Q_0}{RC}$$

How does the current change with time?

To find this, use loop rule w/idea we had @ very end:



positive current comes from <u>losing</u> charge from ⊕ cap plate

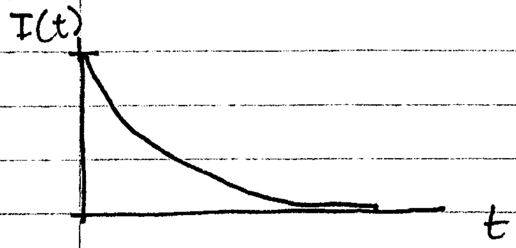
$$\Rightarrow I(t) = -\frac{dQ}{dt}$$

Substitute this into loop rule $\Longrightarrow$ "differential equation" for Q: $Q(t) = -\frac{dQ}{dt}RC$

Result of integrating this (and using initial conditions properly for limits) is an <u>exponential decay</u>:

$$Q(t) = Q_0 e^{-t/RC} \quad \text{and so} \quad I(t) = \frac{Q_0}{RC} e^{-t/RC}$$

What does this mean? Let's look at a graph:



max value at $t=0$

decreases rapidly at first, more slowly afterward

The product RC determines how quickly it decreases:

After a time $t = RC$, $I(t)$ and $Q(t)$ have dropped to $\frac{1}{e}$ times their initial values

$$I(t=RC) = I_0 e^{-RC/RC} = I_0 e^{-1} = \frac{I_0}{e}$$

$\frac{1}{e} \approx 0.37$ so rule of thumb is that after $t = RC$, current/charge has dropped to 37% of its initial value

If you want to discharge <u>fast</u> need small RC

If you want to store lots of energy in the capacitor, need big C

$\Rightarrow$ want small R to discharge a lot of energy rapidly!

Also <u>initial</u> current $= V_0/R$ determined entirely by R, V_0

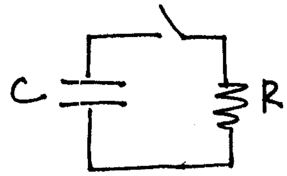
Defibrillator: 100 μF cap,

Resistivity ~~of~~ human body: very high when dry — so need to bring resistance down to get rapid discharge

any EMT's in class?

Remember resistance $R = \frac{\rho L}{A}$ so make paddles on defib large

(Note to self: <u>initial</u> current $= \frac{Q_0}{RC}$ just as if resistor was driven by battery w/ $\mathcal{E} = Q_0/C$)



Sample problem

Goal: times it takes to reach various voltages

Principle: exponential decay of charge on capacitor $\Rightarrow$ exp decay of V_{cap} too

$$V_{cap}(t) = \frac{Q(t)}{C} = \frac{Q_0}{C} e^{-t/RC} = V_0 e^{-t/RC}$$

(a) $V_{cap} = 3.7\ V$

Could recognize this as the 37% rule of thumb from before: V_{cap} has dropped to 37% of its initial value of 10 V.

Do it explicitly:

$$3.7\ V = 10.0\ V e^{-t/(10\ \Omega)(1.0\ F)}$$

note $1\Omega \cdot 1F = 1s$ which you'll show on HW

$1\Omega = 1V/1A \quad 1F = 1C/1V$

Need t: divide both sides by 10 V

$$\left(\frac{3.7\ V}{10.0\ V}\right) = e^{-t/10\ s}$$

Take ln of both sides:

$$\ln(0.37) = \ln(e^{-t/10s}) = -\frac{t}{10s}$$

$$\ln(0.37) = -1 \Rightarrow -1 = -\frac{t}{10s}$$

$$10s = t$$

(b) $V_{cap} = 0\ V$ amounts to asking: does the decay ever get all the way to zero? Do we ever have $e^{-t/RC} = 0$?

Not exactly... graph shows function never quite gets to zero.

Instead ask what is good enough? $V_{cap} = 0.01\ V$ (0.1% of original)

Then $1.0\times10^{-2}\ V = 10\ V e^{-t/RC}$

$$1.0\times10^{-3} = e^{-t/RC}$$

$$\ln(1.0\times10^{-3}) = \ln(e^{-t/RC})$$

$$-6.9 = -\frac{t}{RC} \Rightarrow t = 6.9RC = 69\ s$$

A capacitor with a capacitance of 1 F is charged to 10 V and then allowed to discharge through a 10 Ω resistor. How long does it take for the capacitor voltage to reach 3.7 V? How long does it take for the capacitor to discharge completely?

initial $V_{cap} = 10\text{ V}$, then decreases

Need time $t = RC$ for Voltage to fall to 37% of its initial value

$$V_{cap}(t) = \frac{Q(t)}{C} = \frac{Q_0 e^{-t/RC}}{C} = V_0 e^{-t/RC}$$

Want t:

$$\ln\left(\frac{V_{cap}(t)}{V_0}\right) = \ln\left(e^{-t/RC}\right)$$

$$\ln\left(\frac{V_{cap}(t)}{V_0}\right) = -\frac{t}{RC}$$

$$\ln\left(\frac{3.7\text{ V}}{10\text{ V}}\right) = -\frac{t}{(10\Omega)(1.0\text{F})}$$

$$-1 = -\frac{t}{10\text{s}}$$

$(1\Omega)(1\text{F}) = 1\text{s}$

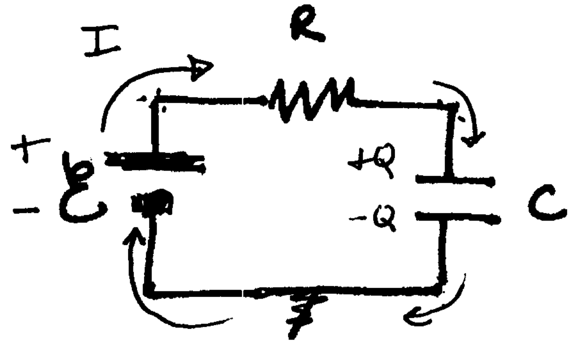
$V = IR \Rightarrow 1\text{V} = (1\text{A})(1\Omega)$

$Q = CV \Rightarrow 1\text{C} = (1\text{F})(1\text{V})$

$$10\text{s} = t$$

When is $V_{cap} = 0$? Only when $e^{-t/RC} = 0$

A circuit consists of a battery with emf $\mathcal{E}$ in series with a capacitor and a resistor. The switch is initially open and the capacitor is not charged. Then the switch is closed. How do the voltage across the capacitor V_{cap} and the current in the resistor I_R change as time goes on?



just as with resistor ckts at every instant, I is same all around circuit

1. V_{cap} will increase and I_R will increase.
2. V_{cap} will increase and I_R will decrease.
3. V_{cap} will decrease and I_R will increase.
4. V_{cap} will decrease and I_R will decrease.

amount of I changes in time BUT same all around @ any inst

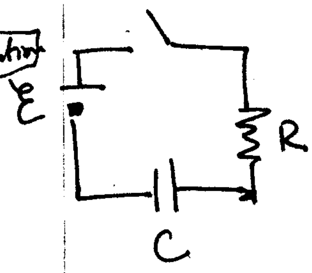
$$\mathcal{E} - I(t)R - V_{cap}(t) = 0$$

$$\frac{\mathcal{E} - V_{cap}(t)}{R} = I(t)$$

11:50

long conversation here

What about charging?



Capacitor is initially uncharged, then close switch

$\boxed{CT}$ $V_{cap} \uparrow$, $I \downarrow$

Why?

Loop rule again!

$\mathcal{E} - I(t)R - V_{cap}(t) = 0$

(why do we subtract $V_{cap}(t)$? current will charge the plate next to ⊕ terminal ⊕, so crossing V_{cap} from ⊕ to ⊖ — losing potential)

V_{cap} increases: we know we start at $V_{cap}(t=0) = 0$ b/c uncharged

eventually reach maximum value (what? $\mathcal{E}$) when fully charged (no more current flows)

As V_{cap} increases, $I(t)R$ decreases so as to keep Loop rule true

Initial value of $I(t=0)$? $I(t=0) = \frac{\mathcal{E}}{R}$ b/c $V_{cap}(t=0) = 0$

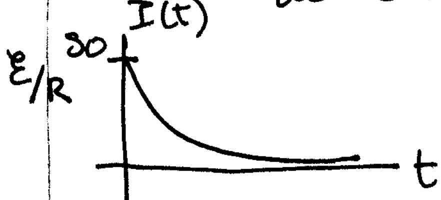
Final value of I? Zero

In fact expect exactly same $I(t)$ as discharging:

initially $\mathcal{E}$ drives current

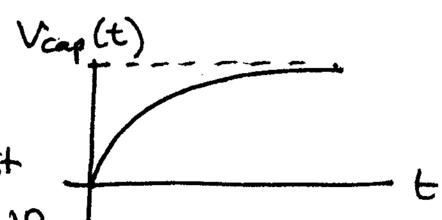
as V_{cap} increases, $\mathcal{E} - V_{cap}(t)$ is driving current → decreasing!

so



and thus $V_{cap}(t)$ must be $\mathcal{E} - I(t)R$

so just same shape flipped over



Mathematically: follow similar procedure of using relnship between Q & I in Loop rule, but now current is adding charge to capacitor

$\Rightarrow I(t) = \dfrac{dQ}{dt}$

Turns out to be easier to find $I(t)$ first

$\rightarrow I(t) = I_0 e^{-t/RC} = \dfrac{\mathcal{E}}{R} e^{-t/RC}$ same! and same time constant

Then $V_{cap}(t) = \mathcal{E} - I(t)R = \mathcal{E}(1 - e^{-t/RC})$ same time const, incr.

$Q(t) = CV_{cap}(t) = C\mathcal{E}(1 - e^{-t/RC})$

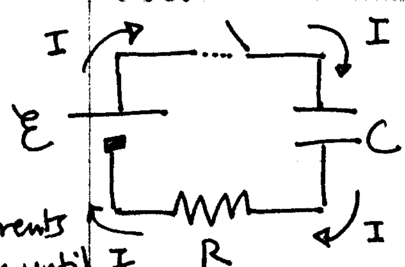
Use this equation the same way as the other one, so problems are very similar

This time rule of thumb gives us at $t = RC$, current is 37% of initial current, but voltage is $(1 - e^{-1})\mathcal{E} \cong (1 - 0.37)\mathcal{E} = 0.63\,\mathcal{E}$

Two time constants: $t = 2RC \Rightarrow (1 - e^{-2})\mathcal{E} \cong 0.86\,\mathcal{E}$

slows down!

Does it matter if instead we have

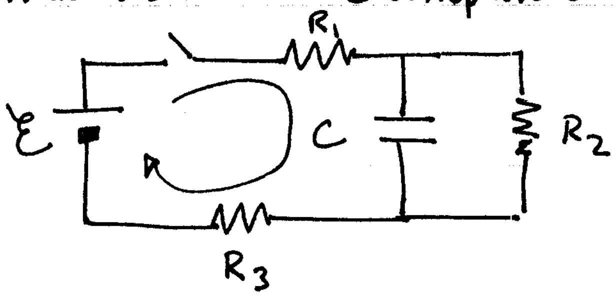


(no currents shown until after ①)

No! Current is same everywhere around ckt once switch is closed. Varies in time, but at any instant have same current throughout.

(Note: switch is also like a capacitor but with negligibly small capacitance, so current flows only for briefest instant in open ckt config)

What about more complex circuits?



When first close switch, capacitor is uncharged
$\rightarrow V_c = 0 \rightarrow$ it behaves like a short circuit
and all current travels down its branch
(C and R_2 are in parallel
if $V_c = 0$ must also have $V_{R_2} = 0$
this can only happen if $I_{R_2} = 0$)

How much current is drawn from battery initially?
Loop rule for left loop:

$$\mathcal{E} - I_{batt}R_1 - I_{batt}R_3 = 0 \qquad V_{cap} = 0 \quad V_{R_2} = 0$$

$$\Rightarrow I_{batt} = \frac{\mathcal{E}}{R_1 + R_3}$$

How fast does it charge?
Complicated: but at initial instant, like $R = R_1 + R_3$

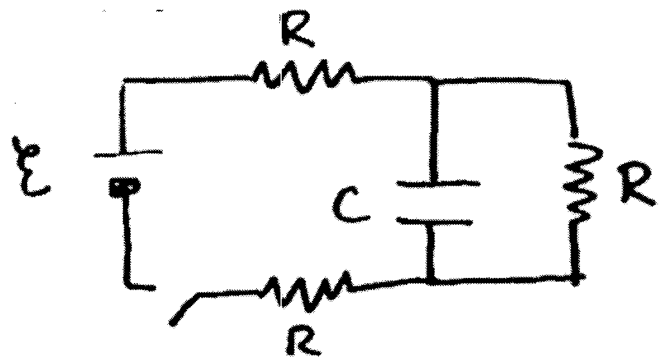
$$\Rightarrow \tau = RC = (R_1 + R_3)C$$

After a very long time, C is fully charged
What is it's final voltage? $V_{cap} < \mathcal{E}$ b/c current still goes around outer loop
Must have $V_{cap} = V_{R_2}$ so must be a current there!
Once fully charged, no current goes down C ("open ckt")
$\Rightarrow$ current only around outer loop
Find that current, use it to find $V_{R_2} \Rightarrow V_{cap}$

$$\mathcal{E} - IR_1 - IR_2 - IR_3 = 0 \Rightarrow I = \frac{\mathcal{E}}{R_1 + R_2 + R_3}$$

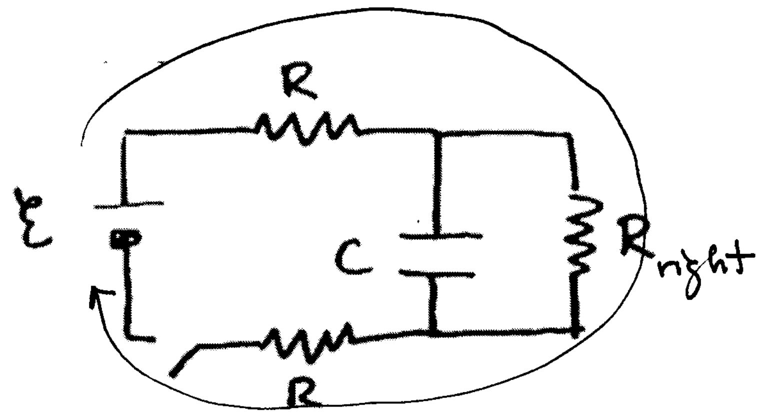
$$V_{R_2} = IR_2 = \mathcal{E}R_2/(R_1 + R_2 + R_3)$$

In the circuit shown, the capacitor is initially uncharged. Immediately after the switch is closed, what is the voltage across the capacitor, V_C?



1. $V_C = \mathcal{E}$
2. $V_C = \mathcal{E}/2$
3. $V_C = \mathcal{E}/3$
4. $V_C = \mathcal{E}/4$
5. $V_C < \mathcal{E}$ but not sure how much less.
6. $V_C = 0$.

In the circuit shown, a long time after the switch is closed, what is the voltage across the capacitor, V_C?



No current in C branch
Current only flows around outside loop

V_{cap} must equal $V_{R_{right}}$

1. $V_C = \mathcal{E}$
2. $V_C = \mathcal{E}/2$
3. $V_C = \mathcal{E}/3$
4. $V_C = \mathcal{E}/4$
5. $V_C < \mathcal{E}$ but not sure how much less.
6. $V_C = 0$.

C is "open circuit" at this point (when fully chgd)

$$\left.\begin{aligned} V_{top} &= IR \\ V_{right} &= IR \\ V_{bottom} &= IR \end{aligned}\right\} \quad \mathcal{E} - 3IR = 0 \quad \Rightarrow I = \frac{\mathcal{E}}{3R}$$

$V = \frac{\mathcal{E}}{3}$ for any resistor