Connecting meaning and math

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Our goal: Multilingual fluency

- Representations ("languages"): mathematics, words, graphs, diagrams
- Experts use multiple representations extensively
- Experts check for consistency between representations and with intuition
- Students find this challenging!

IPLS policy recommendations

- Professional societies also recommend developing “quantitative skills”

HHMI-AAMC

Competency E1

Apply quantitative reasoning and appropriate mathematics to describe or explain phenomena in the natural world.

Learning Objectives:

1. Demonstrate quantitative numeracy and facility with the language of mathematics.

   Examples:

   - Express and analyze natural phenomena in quantitative terms that include an understanding of the natural prevalence of logarithmic/exponential relationships (e.g., rates of change, pH).
   - Explain dimensional differences using numerical relationships, such as ratios and proportions.
   - Use dimensional analysis and unit conversions to compare results expressed in different systems of units.

National Academies/BIO 2010

• Physics principles are critical to both biology and measurement technology

• “These principles provide a simple context in which to learn the relationships between observations and mathematical modeling.”

Recommendation #1.3

IPLS policy recommendations

- Professional societies recommend developing “quantitative skills”
- BUT recommendations are vague
- We need clearer definitions of what students should be able to do, so that we can
  - design appropriate tasks
  - assess if students are making progress

Connecting meaning and math

- We want students to connect mathematical representations with a physical picture of what is happening.
- How do we design problems to do this well?
  - Example 1: Pitfalls of problem-solving strategies
  - Example 2: Coordinating biological meaning with mathematics

Pitfalls of Algorithmic Strategies

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- Textbook provided the following strategy: Model, Visualize, Solve, Assess
- Task: On a hot 35 C day, you perspire 1.0 kg of water during your workout.
  - What volume is occupied by the evaporated water?
  - By what factor is this larger than the volume occupied by the liquid water?

Textbook Solution

- Model: Assume the evaporated water is an ideal gas with a molar mass of 18 *g/mole*. Assume the pressure is 1atm=101.3 *kPa*
- Visualize: We are given T=35 *C* +273=308 *K.* n=1000 *g*(1 *mole*/18 *g*)=55.6 *mol.*
- Solve: (a) $pV=nRT \rightarrow V=1.4\ m^3$
  (b) In the liquid state, $\rho=1000 kg/\ m^3$
  (A simple calculation yields a factor of 1400)
- Assess: Gases really do take up a lot more volume than the equivalent mass of liquid.

Pitfalls of Algorithmic Strategies

- The difficulty is that the answer in the solution manual fails to treat the problem intuitively – to *tell the story of the problem.*
- The steps of the strategy are gone through formally – but without leveraging sense-making intuitions about molecules or even about the nature of gases.

Coordinating physical and biological meaning with math

Coordinating physical and biological meaning with math

- Earthworms absorb oxygen through their skin, but use oxygen throughout their volume.
- Given that the skin of the worm can absorb oxygen at a rate of 0.24 μmole/(cm^2*hr) and that the worm uses oxygen at a rate of 0.98 μmole/(g*hr), **how is the size of the worm constrained?**

Worm Problem allows students to

- Build a simple model of a physical and biological system.
- Tie structural features to organismal function.
- Explore physical and biological implications of quantitative relationships.
- Recognize the biological implications of **competing** functional dependences (e.g., surface vs. volume).

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- Advised students (on study guide sheet and individually in person) to study by "translating" a list of key equations

Task idea: explicit translation

- Advised students (on study guide sheet and individually in person) to study by “translating” a list of key equations, including suggesting an example
- Many students said they found this helpful
- Similar spirit to Etkina & van Heuvelen “Equation Jeopardy”

Final exam short answer

"Choose one of the three equations below. Explain in words the physical relationship described by the equation, explain what each symbol in the equation represents (including units), and describe what kind of situations this equation applies to. If appropriate, include a sketch to clearly define the quantities in the equation/your explanation."

Future improvements

- Give explicit tasks along these lines on the homework
- Ask students to devise a specific problem (Equation Jeopardy approach)

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